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## Where to dig for gold? Simple 'tricks' to help find new universality classes

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### Physics ~ Universal behaviour + O(model specific details) \*

\*Asymptotic series: leading terms can be different

## Mathematical definition of universality

**Universality Class** = Fixed point (**FP**) of renormalisation group

(**RG**) coarse graining procedure

**Universal behaviour** = Properties quantifiable by RG FP

**Relevant terms in model equations** = Terms that affect RG FP

**Irrelevant terms** = Terms that do not affect RG FP

# **Renormalisation group**

A mathematical, coarse-graining procedure that incorporates

fluctuations to quantify how model parameters are modified

at larger length scales



RG transformation





#### Categorisation: chemical elements vs. universality classes



Ising model:  $H = \sum_{\langle i, j \rangle} J s_i s_j$ 



Symmetries & conservation laws (SCL)  $\rightarrow$  UC

Atomic no.  $\rightarrow$  elements

# Why universality classes ~ gold?

#### UCs are eternal

Its utility tends to go up with time

#### -Kardar-Parisi-Zhang model as an example

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PHYSICAL REVIEW LETTERS

3 MARCH 1986 ARTICLE

Dynamic Scaling of Growing Interfaces

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Mapping two-dimensional polar active fluids to two-dimensional soap and one-dimensional sandblasting

**OPEN** 

DOI: 10.1038/ncomms12215

Leiming Chen<sup>1</sup>, Chiu Fan Lee<sup>2</sup> & John Toner<sup>3,4</sup>

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PHYSICAL REVIEW LETTERS 128, 070401 (2022)

Emergent Kardar-Parisi-Zhang Phase in Quadratically Driven Condensates

Oriana K. Diessel<sup>(a)</sup>,<sup>1</sup> Sebastian Diehl,<sup>2</sup> and Alessio Chiocchetta<sup>2</sup> <sup>1</sup>Max-Planck-Institute of Quantum Optics, Hans-Kopfermann-Strasse 1, 85748 Garching, Germany <sup>2</sup>Institute for Theoretical Physics, University of Cologne, Zülpicher Strasse 77, 50937 Cologne, Germany

Directed Random (Miermediate Scc ling weakly asymmetric scaling ASEP Polymers + KPZ equation + TASEP log Gamma Polymer **q** -Semi- discrete polymer TASEP Stochastie Homilton-Jacobi Equations Last passage fixed point, KPZ percolation. Stochastic Burgers Equation Polynuclear Growth Stochastic Reachion-Diffusion Model. KPZ Scaling Equations First passage Ballistic Aggrogation percolation Bactenial Richardson Model Colony Boundaries Eden Model. Slide from J. Quastel (Toronto)

# Simple tricks to identify new UCs

- Start with a model with new symmetries & conservation laws
- 2. Mean-field analysis
- 3. Linear stability analysis
- 4. Linear fluctuating hydrodynamics
- 5. Power counting (zeroth order RG) analysis

Colour wheel used to determine direction of velocity





D Nesbitt, G Pruessner CFL (2021) NJP 23, 043047

# 1. Start with a model with new symmetries & conservation laws: *Polar active fluids as an example*

#### **Polar active fluids**

• Hydrodynamic variables: density  $\rho$  and momentum  $\vec{g}$ 

• Conservation law: Mass conservation

$$\partial_t \rho + \nabla \cdot \mathbf{g} = 0$$
  
 $\partial_t \mathbf{g} = \mathbf{F}$ 

$$\partial_t \rho + \nabla \cdot \mathbf{g} = 0$$
  
 $\partial_t \mathbf{g} = \mathbf{F}$ 

• What is the force **F**?

**Symmetries** 

- Starting with symmetries:
  - Temporal invariance: F does not depend on time
  - Translational invariance: F does not depend on position r
  - Rotational invariance: F does not depend on a particular direction
  - Chiral (parity) invariance: F is not right-handed or left-handed

# **Toner-Tu Equations of Motion (EOM)** $\partial_t \rho + \nabla \cdot \mathbf{g} = 0$ $\partial_t \mathbf{g} + \lambda_1 (\mathbf{g} \cdot \nabla) \mathbf{g} + \lambda_2 (\nabla \cdot \mathbf{g}) \mathbf{g} + \lambda_3 \nabla g^2 = U(\rho, \mathbf{g}) \mathbf{g} - \kappa_1 \nabla \rho - \kappa_2 \mathbf{g} (\mathbf{g} \cdot \nabla \rho) + \mu \nabla^2 \mathbf{g} + \mathbf{f} + \cdots$

#### 2020 APS Onsager Prize







Citation: "For seminal work on the theory of flocking that marked the birth and contributed greatly to the development of the field of active matter."



#### 2. Mean-field analysis

#### Mean-field analysis of polar active fluids





Spatiotemporally homogeneous solutions:



Ordered phase of systems with new SCL
 → New UC for the ordered phase?

2. Critical transition from disordered phase to a new ordered phase
→ New UC for the critical behaviour?

Focus of today

### $MF \rightarrow A$ new phase and a new critical UC



#### 3. Linear stability analysis

### Linear stability analysis

- MF analysis  $\rightarrow$  two spatiotemporally homogeneous phases



Linear stability: disordered phaseOrdered phaseDisordered phase• Start with linearised TT EOM:
$$\partial_t \rho + \nabla \cdot \mathbf{g} = 0$$
, $\partial_t \mathbf{g} = -\alpha_0 \mathbf{g} - \kappa_1 \nabla \delta \rho + \mu_1 \nabla^2 \mathbf{g}$ • Consider homogeneous state with perturbations: $\rho(t, \mathbf{r}) = \rho_0 + \epsilon_\rho \exp(st - i\mathbf{q} \cdot \mathbf{r})$ • Substitution into linearised EOM: $s\epsilon_\rho + i\mathbf{q} \cdot \vec{\epsilon}_g = 0$ , $s\vec{\epsilon}_g = -\alpha_0\vec{\epsilon}_g - \kappa_1i\mathbf{q}\epsilon_\rho - \mu_1q^2\vec{\epsilon}_g$ • Re-write as an eigenvalue problem: $s\Phi = M_D \cdot \Phi$ , $\Phi = (\epsilon_\rho \ \vec{\epsilon}_g)^T$  $M_D = \begin{pmatrix} 0 & -i\mathbf{q} & 0 \\ -i\kappa_1\mathbf{q} & -\alpha_0 - \mu q^2 & 0 \\ 0 & 0 & (-\alpha_0 - \mu q^2) \mathbf{I}_{d-1} \end{pmatrix}$ • Solve for eigenvalues: $s = \begin{cases} -\alpha_0 - \mu q^2 \\ -\frac{\alpha_0 + \mu q^2}{2} \pm \sqrt{\frac{(\alpha_0 + \mu q^2)^2}{4} - \kappa_1 q^2}} & \frac{q \to 0}{2} \begin{cases} -\alpha_0 \\ -\frac{\kappa_1}{\alpha_0}q^2 & Always negative! \end{cases}$ 



- Substitution into linearised EOM
- Re-write as an eigenvalue problem

• Solve for eigenvalues: 
$$s = \left[\frac{\alpha_1^2}{\beta} - 2|\alpha_0|\left(2\kappa + \frac{\alpha_1\lambda}{\beta}\right)\right]q^2 + \mathcal{O}(q^4)$$
 Always positive when  $\alpha_0$  is small!

where 
$$\alpha(\rho_0 + \delta\rho) = \alpha_0 + \alpha_1\delta\rho + \mathcal{O}(\delta\rho^2)$$

#### Linear stability analysis $\rightarrow$ more complicated phase diagram



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#### Phase separation $\rightarrow$ Banding regime

G Grégoire & H Chaté (2004) PRL 92, 025702 E Bertin, M Droz & G Grégoire (2006) PRE 74, 022101

Colour wheel used to determine direction of velocity





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#### Linear stability analysis $\rightarrow$ more complicated phase diagram



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#### No so fast!

Remember that in the ordered phase, the key eigenvalue is

$$s = \left[\frac{\alpha_1^2}{\beta} - 2|\alpha_0| \left(2\kappa + \frac{\alpha_1\lambda}{\beta}\right)\right] q^2 + \mathcal{O}(q^4)$$
  
where  $\alpha(\rho_0 + \delta\rho) = \alpha_0 + \alpha_1\delta\rho + \mathcal{O}(\delta\rho^2)$ 

s

What if  $\alpha_1$  is zero? E.g., collective motion speed goes down with density :

 $\rightarrow \alpha_1$  can be zero and so no instability as all!



#### An example





### **Recovery of ordered-disordered critical point**



Linear stability → An enriched phase diagram with a new critical UC by fine tuning 2 parameters

#### Colour wheel used to determine direction of velocity





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#### 4. Linear fluctuating hydrodynamics

#### Linear fluctuating hydrodynamics at criticality

• Start with linearised TT EOM with noise:

$$\partial_t \rho + \nabla \cdot \mathbf{g} = 0$$
,  $\partial_t \mathbf{g} = -\mathbf{\kappa}_1 \nabla \delta \rho + \mu_1 \nabla^2 \mathbf{g} + \mathbf{f}$ 

• Fourier transformed spatiotemporally

$$\begin{split} \delta\rho(\omega_q,\mathbf{q}) &= \int_{\tilde{\mathbf{r}}} e^{\mathrm{i}\omega t - \mathrm{i}\mathbf{q}\cdot\mathbf{r}} \delta\rho(t,\mathbf{r}) \ ,\\ \mathbf{g}(\omega_q,\mathbf{q}) &= \int_{\tilde{\mathbf{r}}} e^{\mathrm{i}\omega t - \mathrm{i}\mathbf{q}\cdot\mathbf{r}} \mathbf{g}(t,\mathbf{r}) \end{split}$$

- Re-write in matrix form
- Find correlation functions between hydrodynamic fields to find

$$\begin{array}{ll} \langle \rho(\mathbf{r},t)\rho(\mathbf{r}',t)\rangle \propto |\mathbf{r}-\mathbf{r}'|^{2-d} & \langle \mathbf{g}(\mathbf{r},t)\cdot\mathbf{g}(\mathbf{r}',t)\rangle \propto |\mathbf{r}-\mathbf{r}'|^{2-d} \\ \langle \rho(\mathbf{r},t)\rho(\mathbf{r},t')\rangle \propto |t-t'|^{(2-d)/2} & \langle \mathbf{g}(\mathbf{r},t)\cdot\mathbf{g}(\mathbf{r},t')\rangle \propto |t-t'|^{(2-d)/2} \end{array}$$

Linear fluctuating hydrodynamics  $\rightarrow$  power-law correlations among hydrodynamic fields at criticality



#### 5. Power counting (zeroth order RG) analysis

#### Is it a new UC? $\mathbf{r} \mapsto e^{\ell} , t \mapsto e^{z\ell} t , \delta \rho \mapsto e^{\chi \ell} \delta \rho , \mathbf{g} \mapsto e^{\chi \ell} \mathbf{g}$

- Remember the scaling exponents from the linear theory: z = 2,  $\chi = \frac{2-d}{2}$
- Use them to ascertain 1) the upper critical dimension, and 2) relevant nonlinear terms
- Here, upper critical dimension is 4, and some of the relevant terms are

$$\partial_t \mathbf{g} + \lambda_1 (\mathbf{g} \cdot \nabla) \mathbf{g} + \lambda_2 \mathbf{g} (\nabla \cdot \mathbf{g}) + \frac{\lambda_3}{2} \nabla (|\mathbf{g}|^2) = -\beta |\mathbf{g}|^2 \mathbf{g} + \dots + \mathbf{f}$$

• No previous models have studied these nonlinearities before!

Power counting  $\rightarrow$  critical model contains nonlinearities never studied before  $\rightarrow$  indicative of a new UC

## Summary: simple tricks to identify new UCs

- New set of symmetries & conservation laws: polar active fluids as an example
- 2. Mean-field analysis  $\rightarrow$  critical point
- 3. Linear stability analysis → enriched phase diagram
- 4. Linear fluctuating hydrodynamics → critical exponents
- 5. Power counting (zeroth order RG) analysis → upper critical dimensions + relevant terms

# Outlook 1

Now that we know where to dig, how do we actually start digging?

Gold standard: Renormalisation group analysis (Wilsonian, fieldtheoretic, functional)









# Outlook 2



5 decades ago, technological relevance, experimental advances, and abundance of novel physics propelled condensed matter physics to become the 'King of Physics' [Martin (2019) Physics Today]

With its relevance to health and life, experimental advances, and abundance of novel physics, I believe biophysics will be the next 'condensed matter physics'

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