Universality in Soft Active Matter

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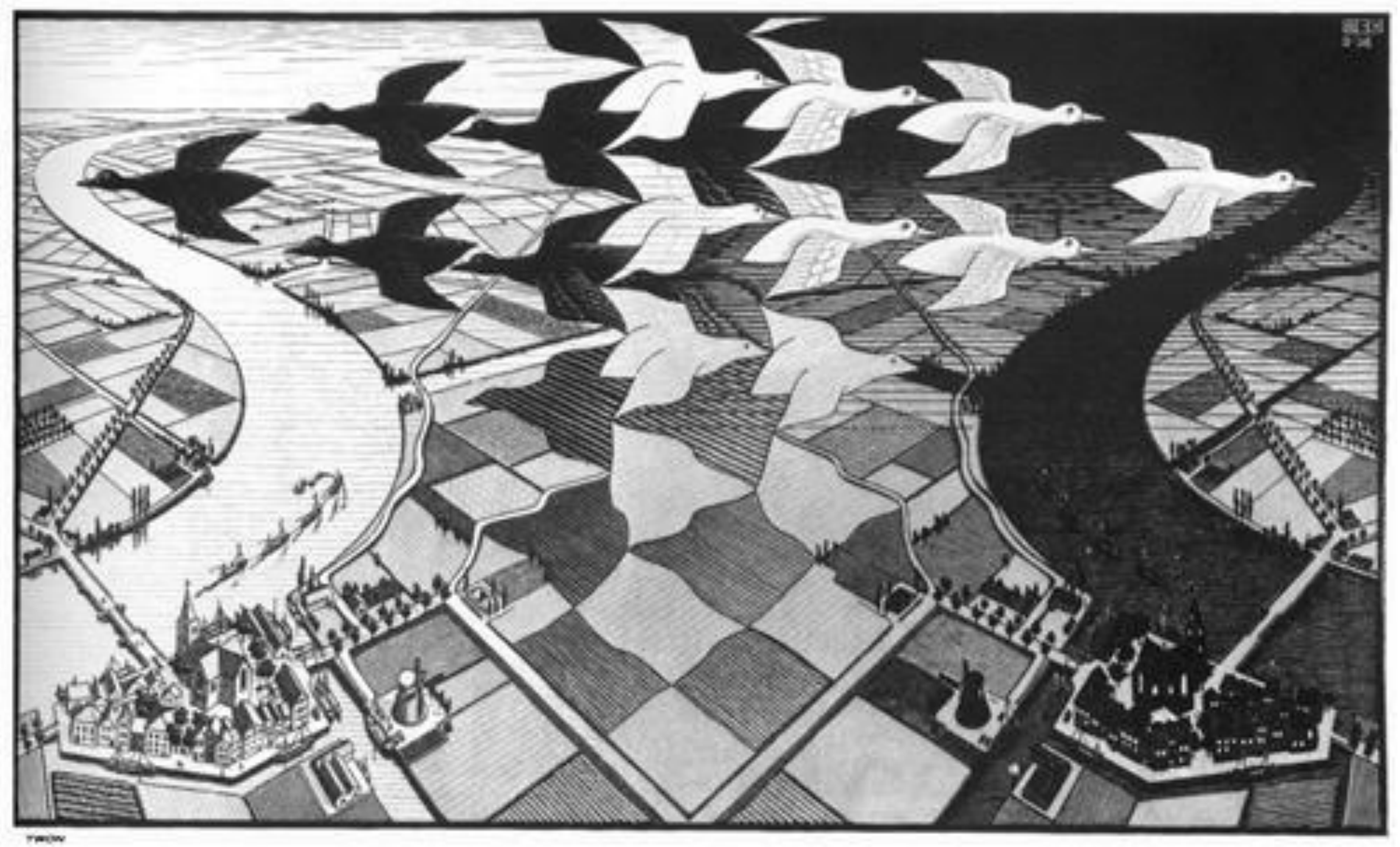
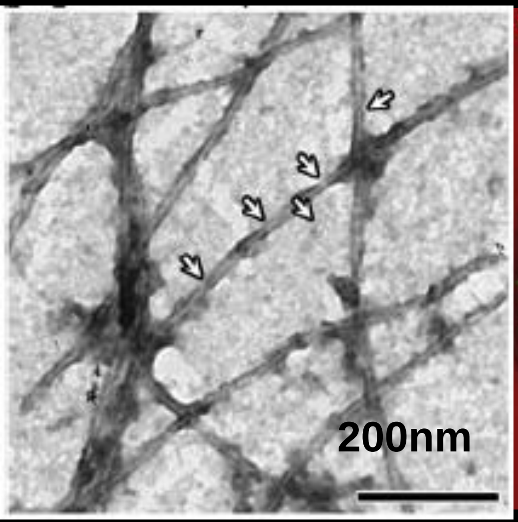


M.C. Escher (1938) *Day and Night*

Universality in soft living matter

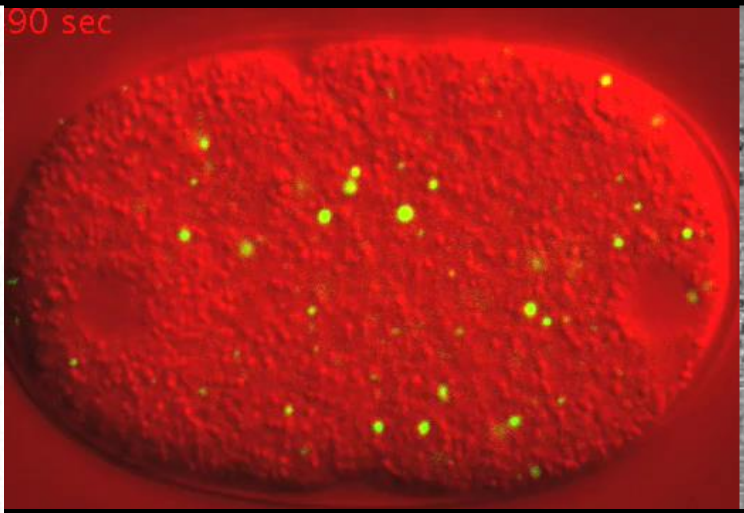
Amyloid fibrils



Equilibrium: length distribution, breakage profile

Non-equilibrium universality?

RNA granules

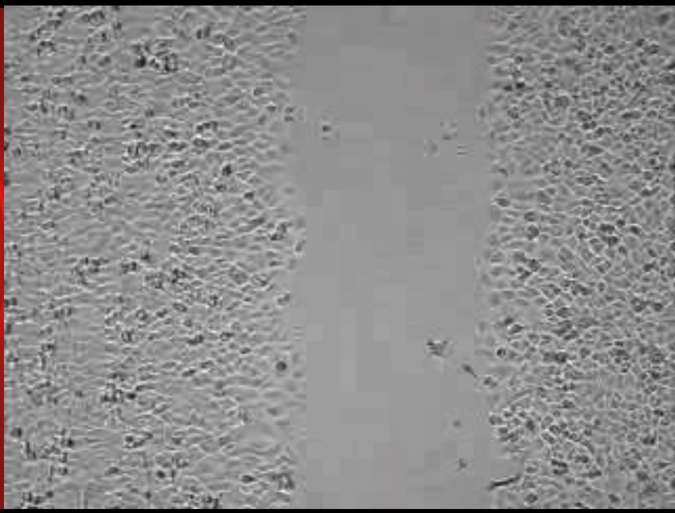


Brangwynne [Hyman Lab]

Equilibrium: Lifshitz-Slyozov scaling & drop size distribution

Non-equilibrium universality?

Active matter



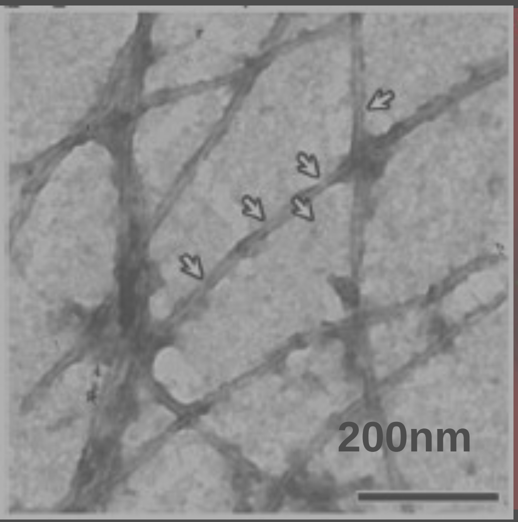
https://www.youtube.com/watch?v=EimBzUSmak8

Fundamentally non-equilibrium

Universality?

Universality in soft living matter

Amyloid fibrils



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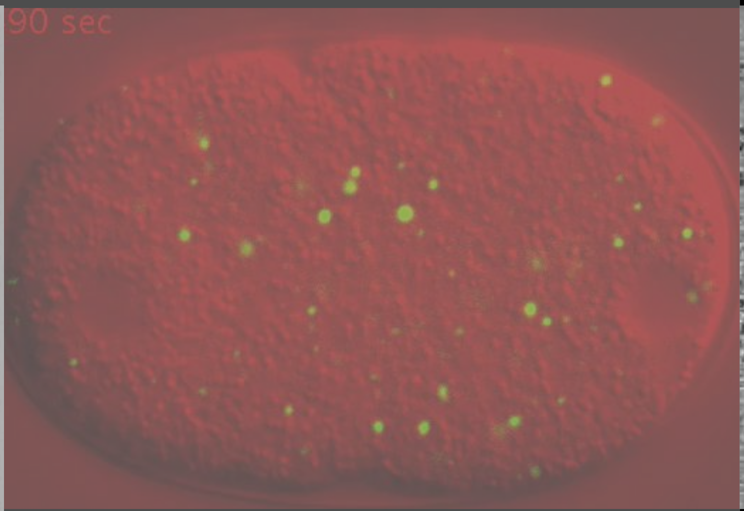

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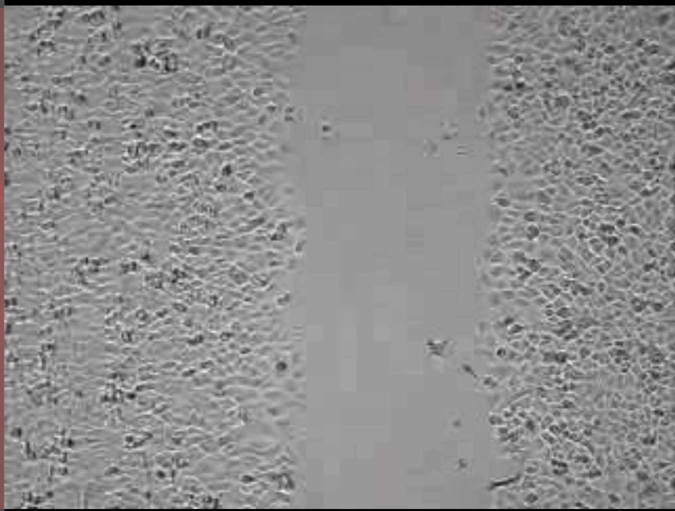

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Fundamentally non-equilibrium

Universality?

Plan

- Minimal system of active particles
  - Motility-induced phase separation
  - Universality in phase separation kinetics
- Modern concept of universality
- Incompressible polar active fluids
  - Universality at criticality
  - Universality in the ordered phase (2D)
- Summary & Outlook

Liquid-gas phase separation

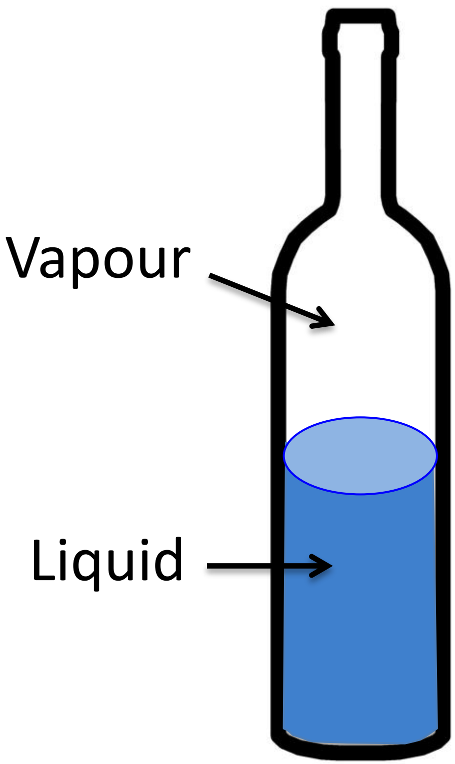
Equilibrium phase separation

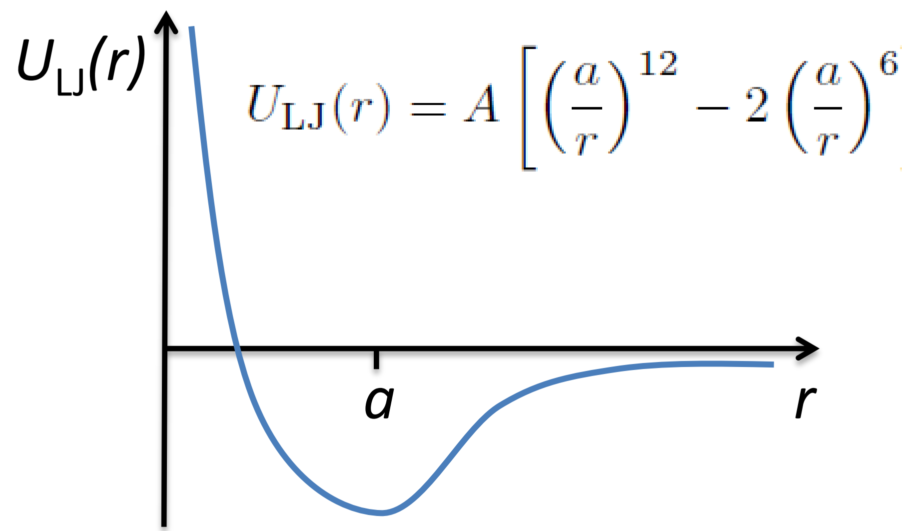
Lennard-Jones liquid:

$$\frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t} = -\frac{1}{\eta}\sum_{j\neq i}\vec{\nabla}_{\mathbf{r}_i} U_{\mathrm{LJ}}(|\mathbf{r}_i - \mathbf{r}_j|) + \sqrt{\frac{2k_BT}{\eta}}\, g_i(t)$$

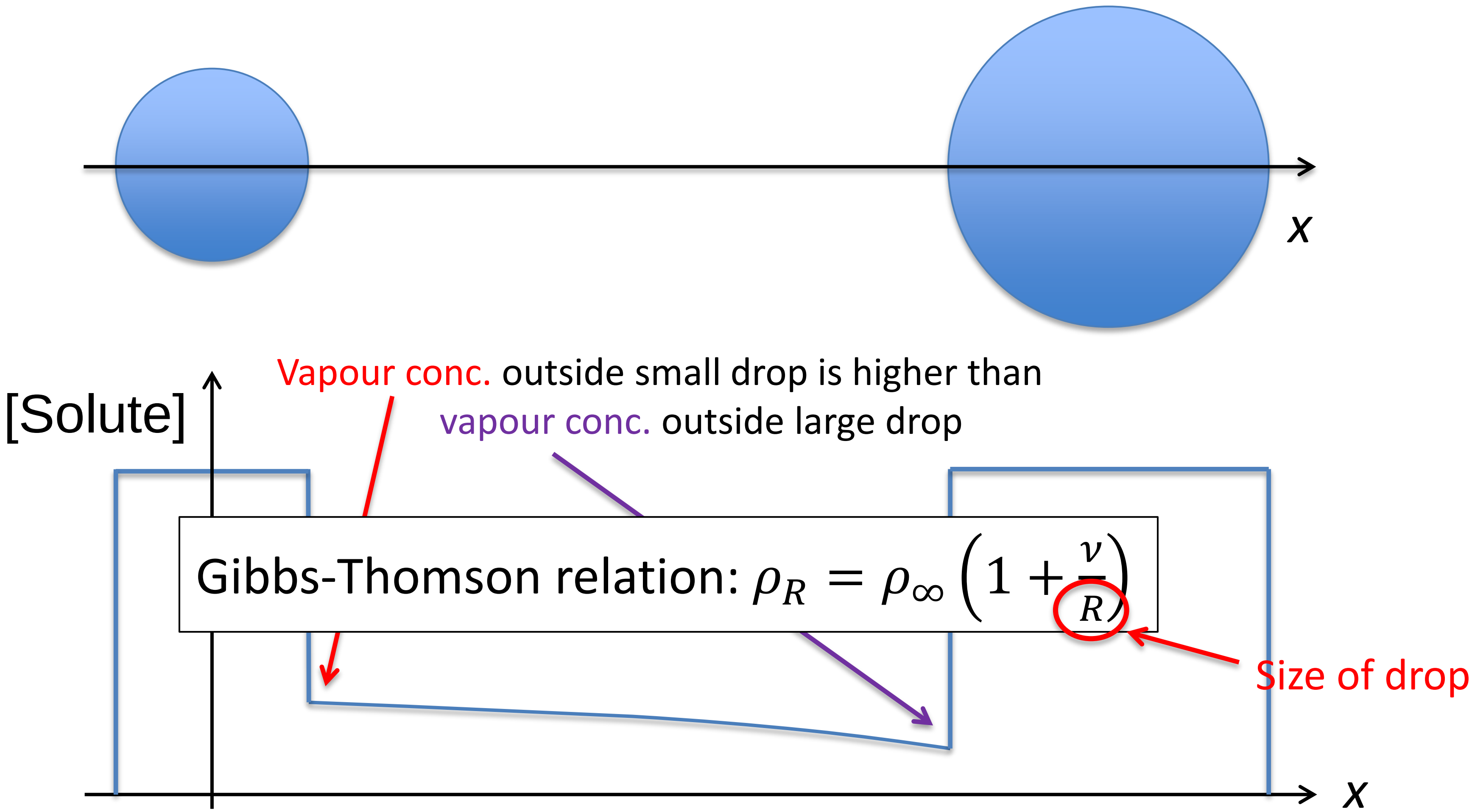
Drag coeff. (η)

Gaussian noise ($g_i(t)$)



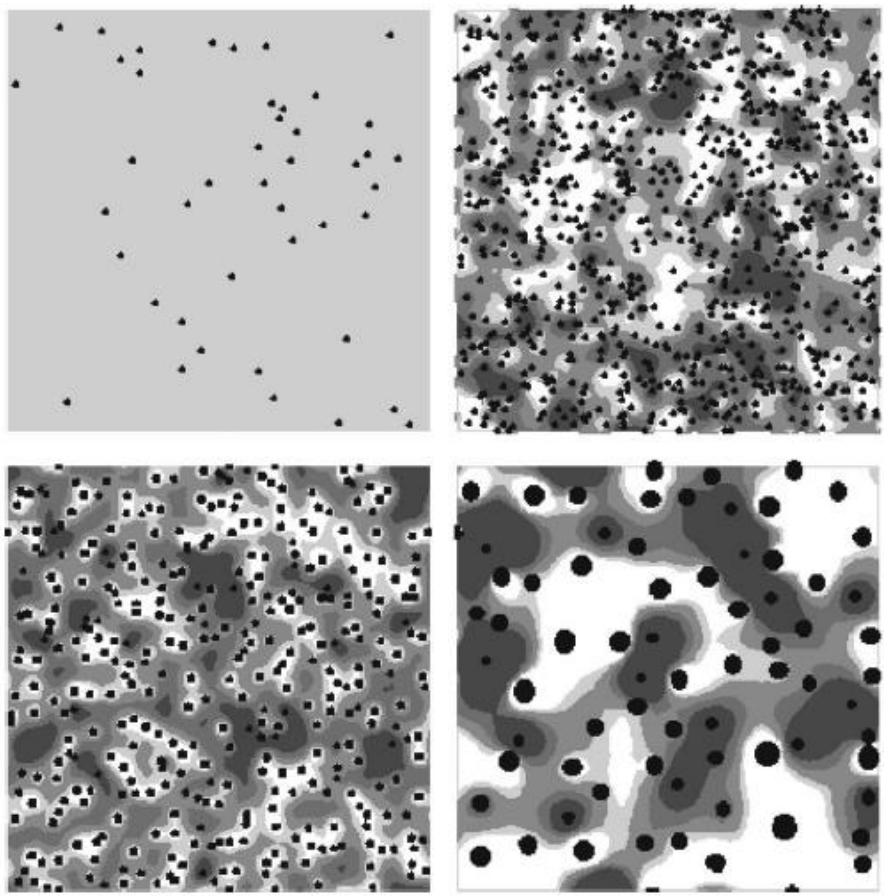


Equilibrium vapour concentration



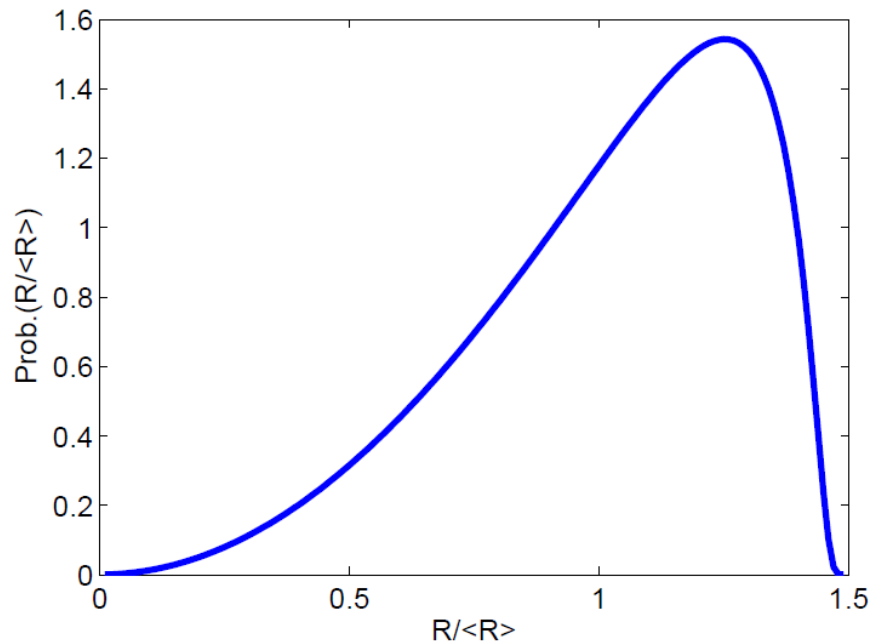
Universality of surface-tension driven droplet growth

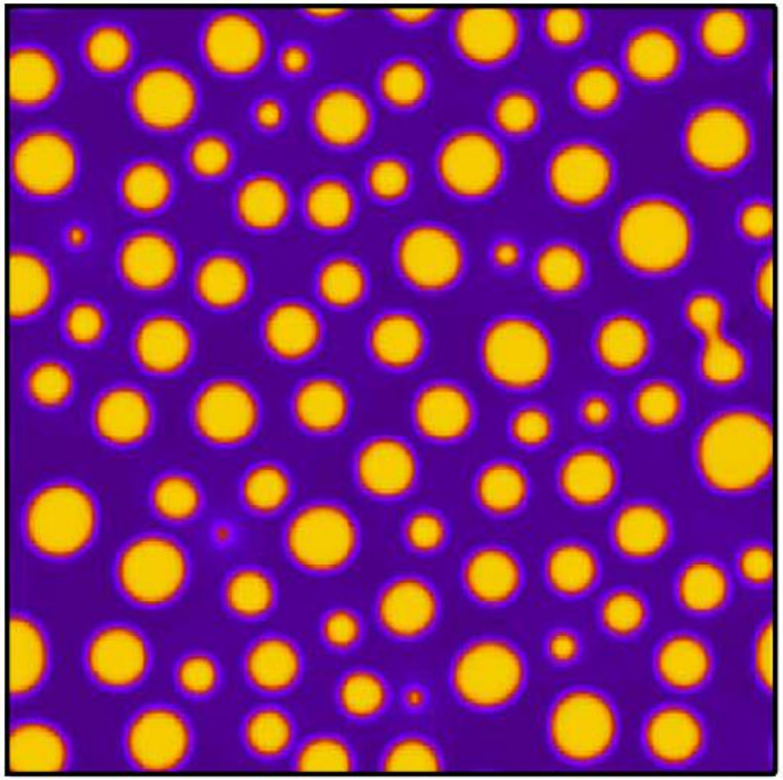
Oswald ripening



Sagui & Grant (1999) PRE

- Lifshitz-Slyozov **universal** growth law: $\langle R(t)\rangle \sim t^{1/3}$
- **Universal** droplet size distribution

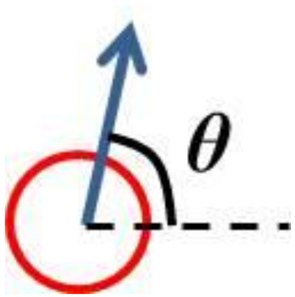




Wittkowski et al. (2013) Nat. Comm.

Motility-induced phase separation

Motility-induced phase separation (MIPS)

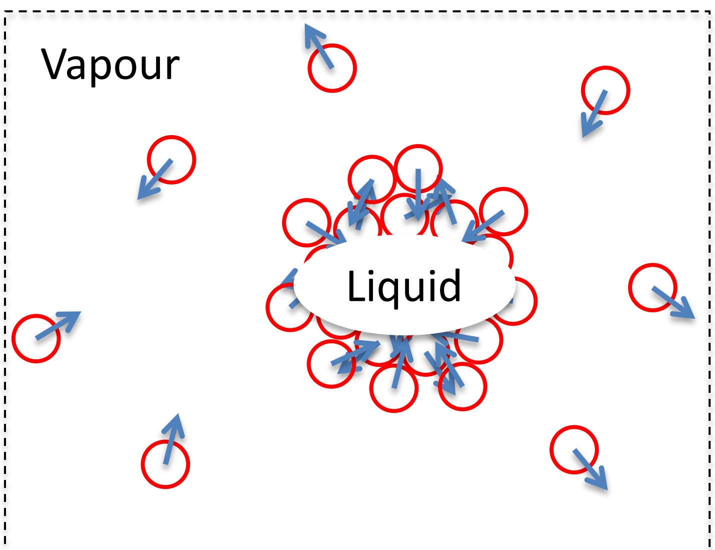


Active force

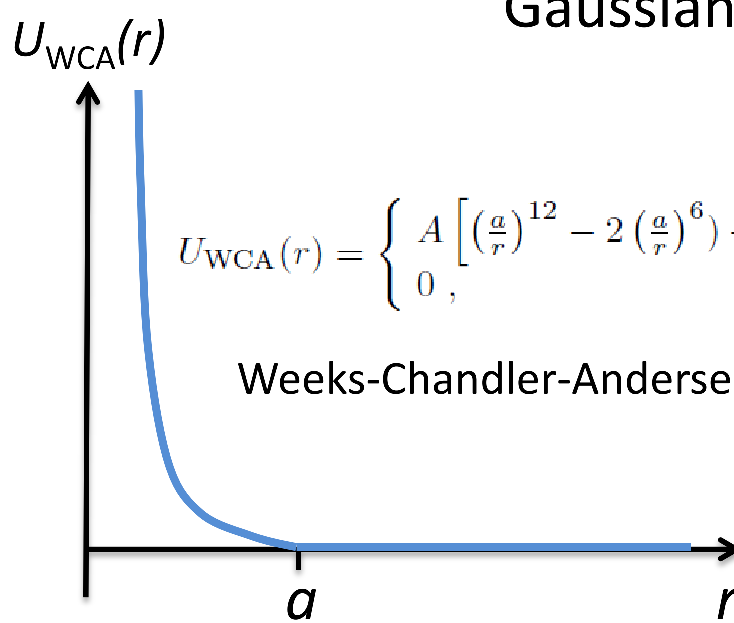
$$\frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t} = -\frac{1}{\eta}\sum_{j\neq i}\vec{\nabla}_{\mathbf{r}_i} U_{\mathrm{WCA}}(|\mathbf{r}_i - \mathbf{r}_j|) + \frac{f_a}{\eta}\mathbf{v}_i$$

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \sqrt{2D_r}\, g_i(t) \quad , \quad \mathbf{v}_i = \cos\theta_i \hat{\mathbf{x}} + \sin\theta_i \hat{\mathbf{y}}$$

Gaussian noise

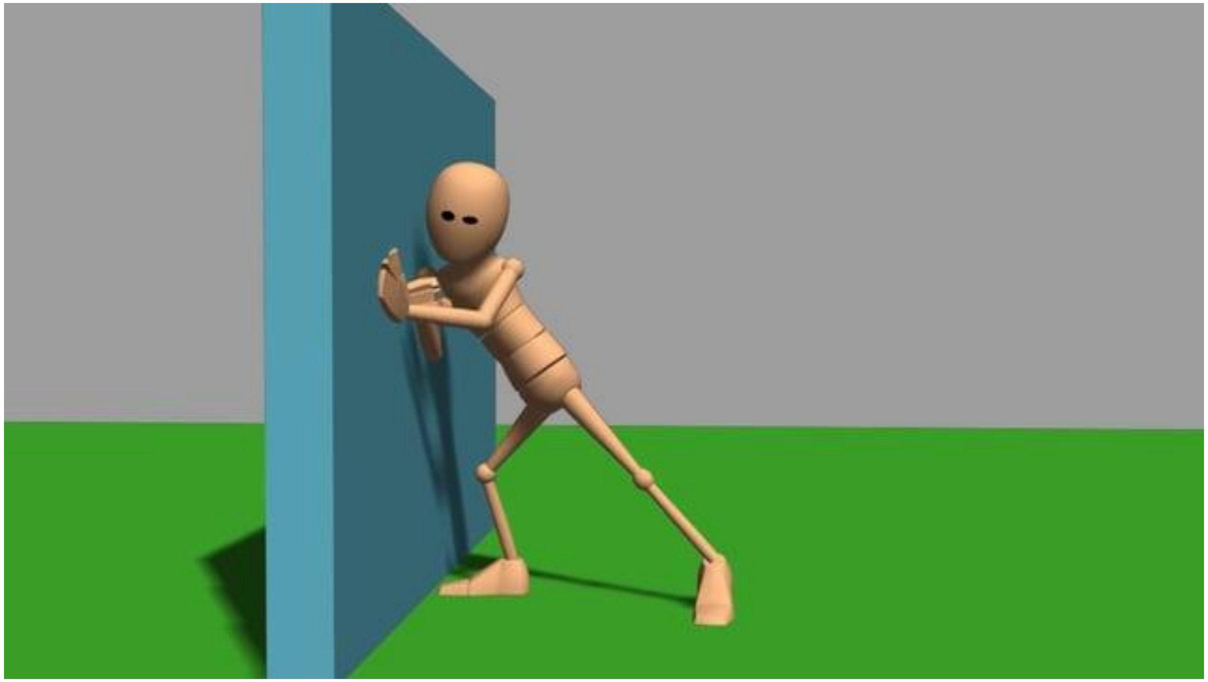


Tailleur & Cates (2008) PRL;
Fily & Marchetti (2012) PRL;
Redner, Hagan & Baskaran (2013) PRL



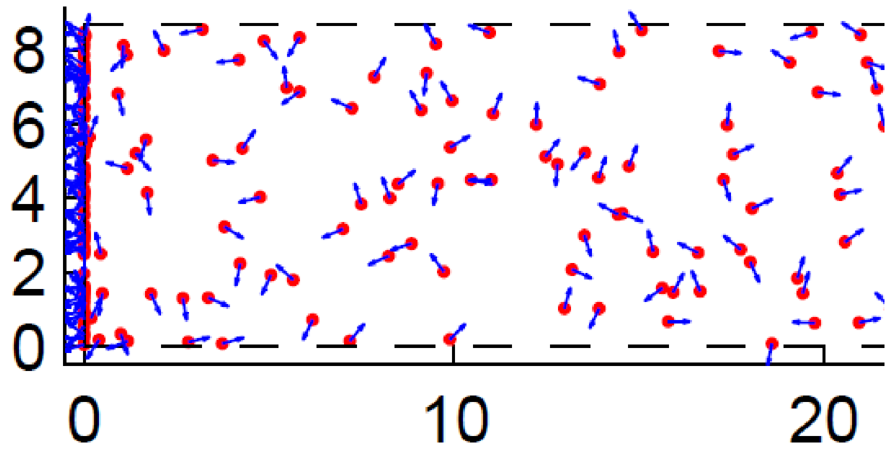
$$U_{\mathrm{WCA}}(r) = \begin{cases} A\left[\left(\frac{a}{r}\right)^{12} - 2\left(\frac{a}{r}\right)^{6}\right) + 1\right] , & \text{if } r < a \\ 0 , & \text{otherwise} \end{cases}$$

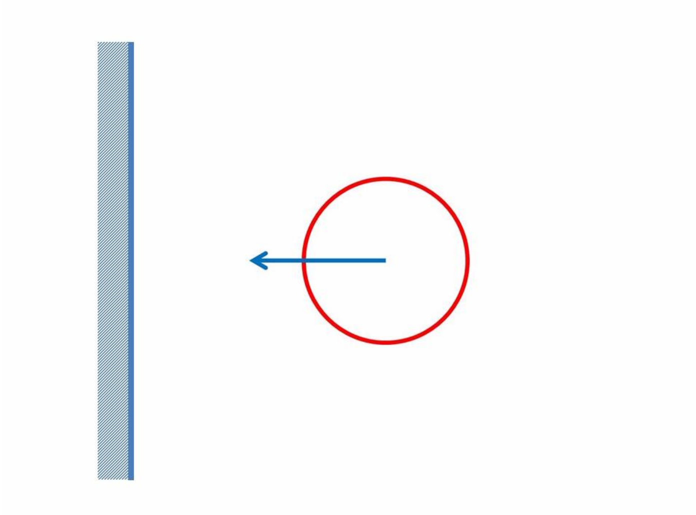
Weeks-Chandler-Andersen potential

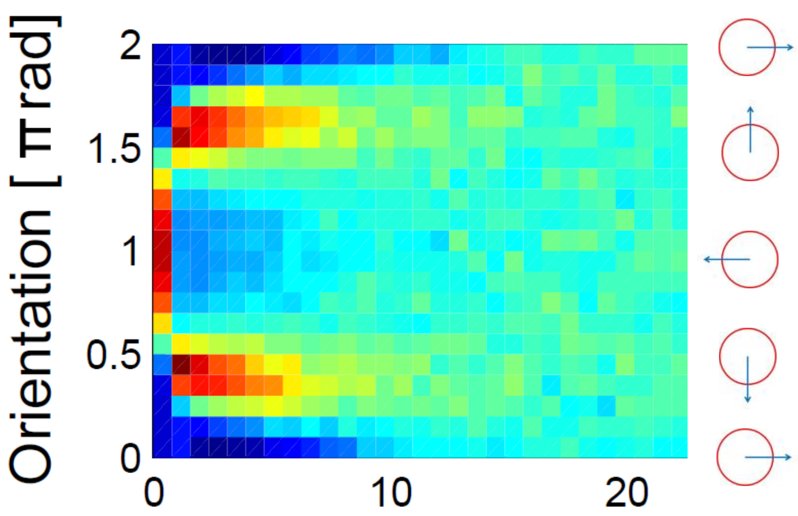


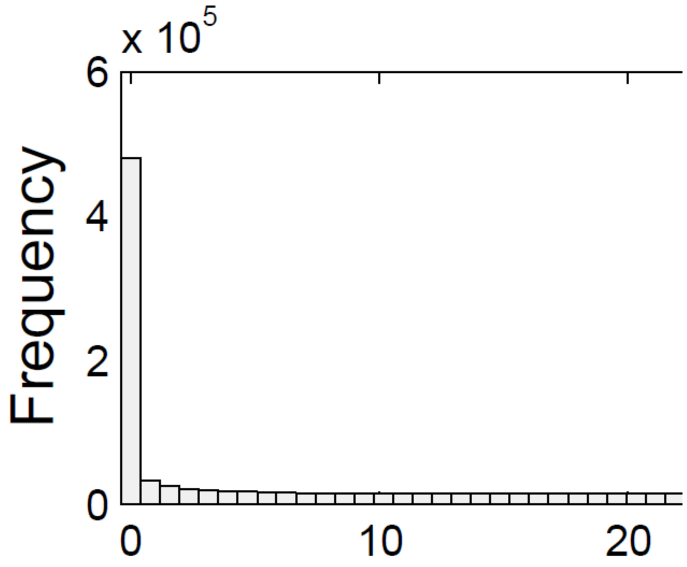
Force balance at the interface in MIPS

Confined active point particles

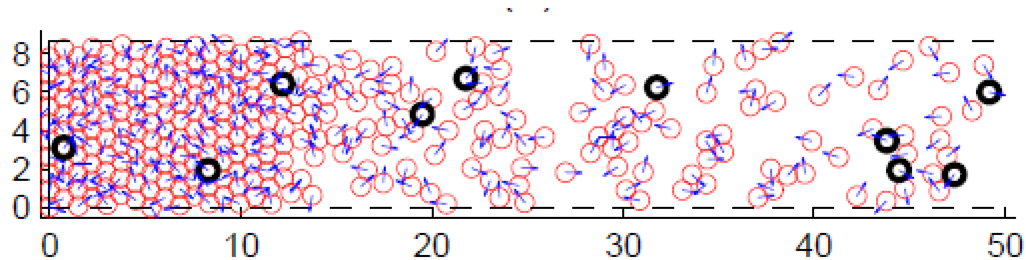


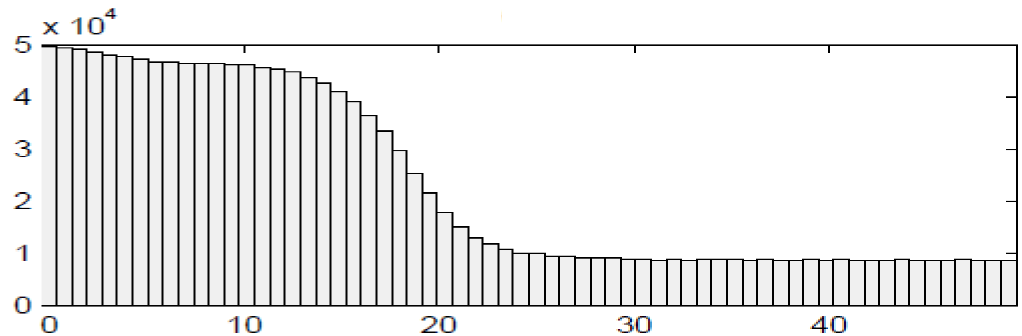




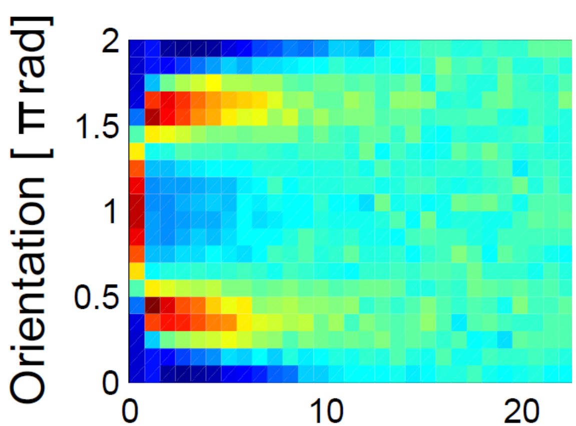


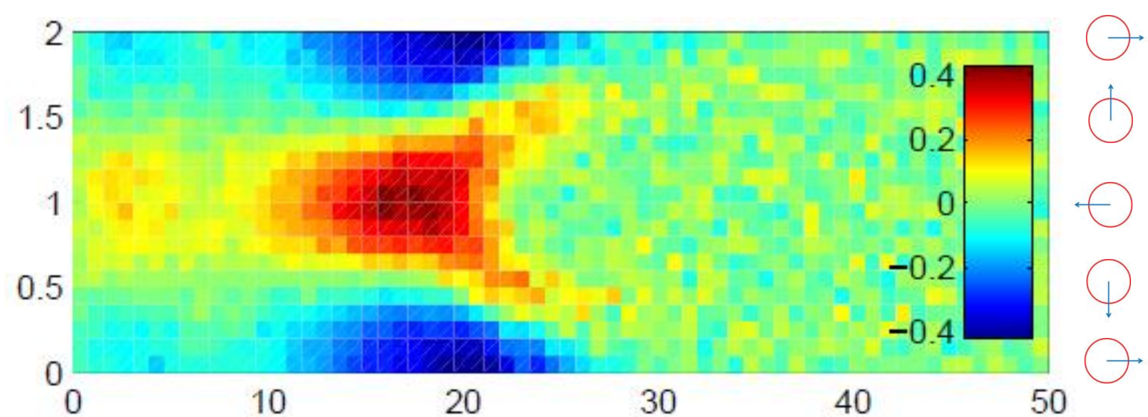
Repulsive active point particles





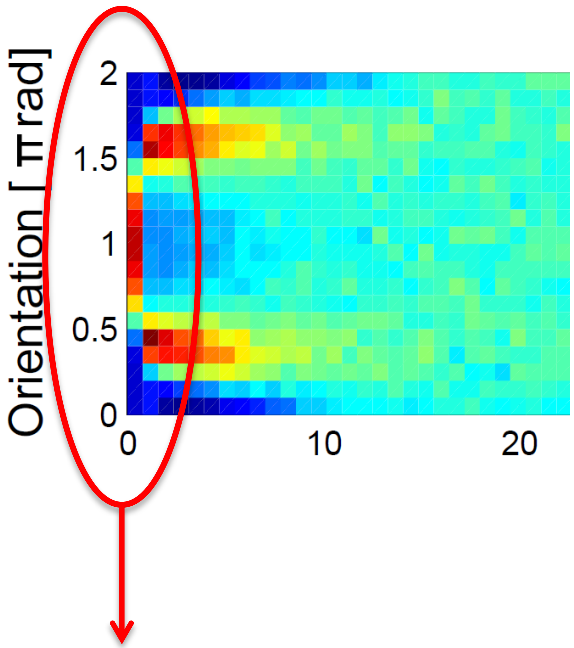
Active point particles





Forces on the wall

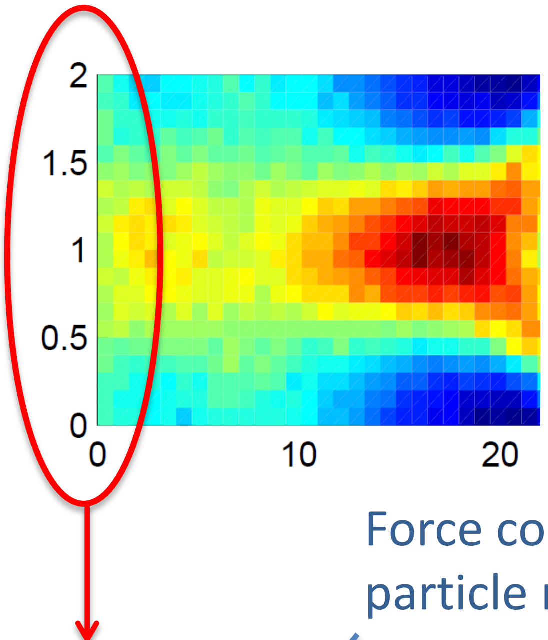
Point particles



$$\frac{\rho_v f_a^2}{2\eta D_r}$$

[Swim pressure:
Takatori, Yan & Brady (2014)
PRL]

Repulsive particles

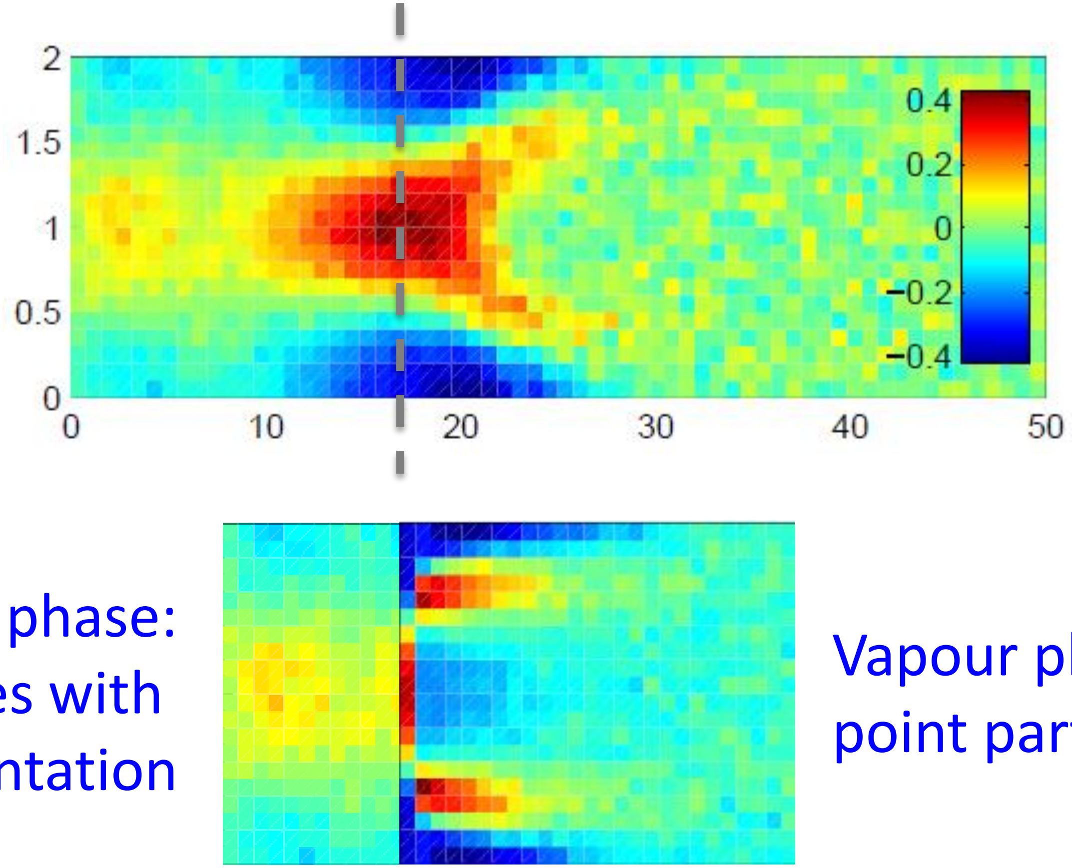


Force coming from particle repulsion

$$2a\rho_c f_a + f_r$$

High active force from the point particles due to orientation anisotropy

Sharp interface model



Condensed phase: particles with isotropic orientation

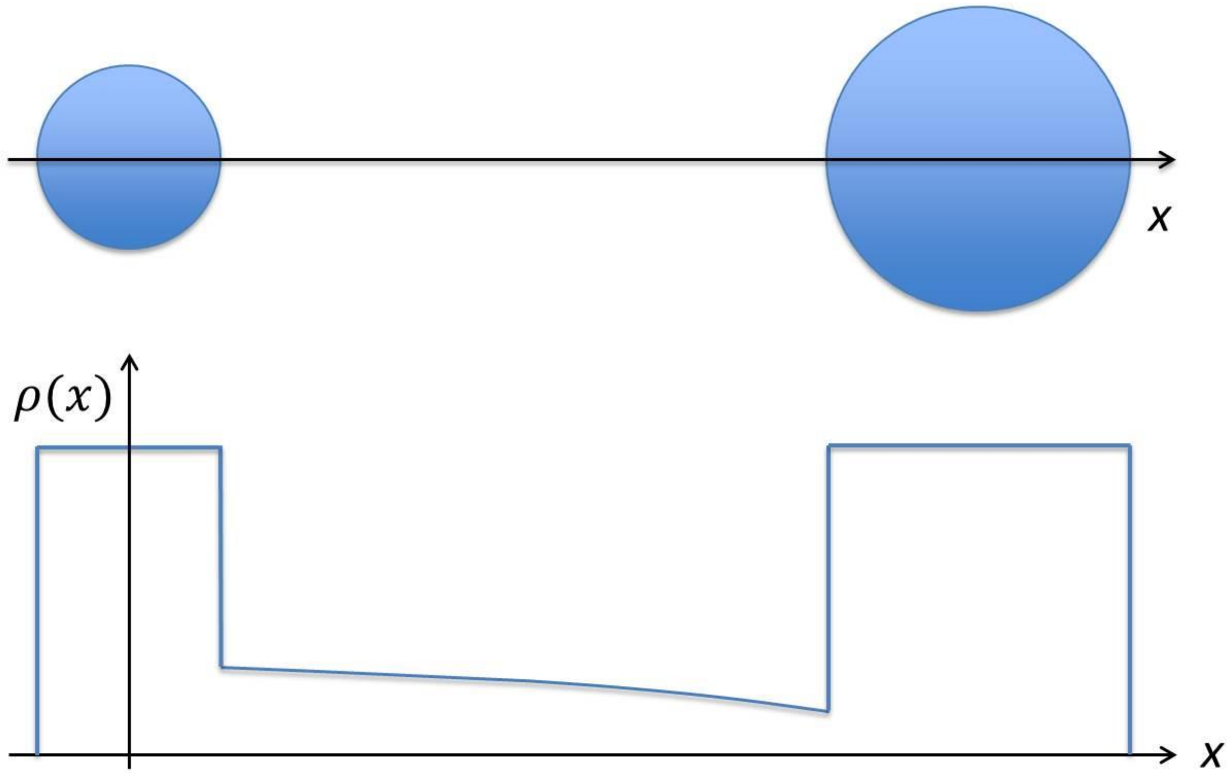
Vapour phase: point particles

$$2a\rho_c f_a + f_r = \frac{\rho_v f_a^2}{2\eta D_r}$$

Min. force for MIPS:

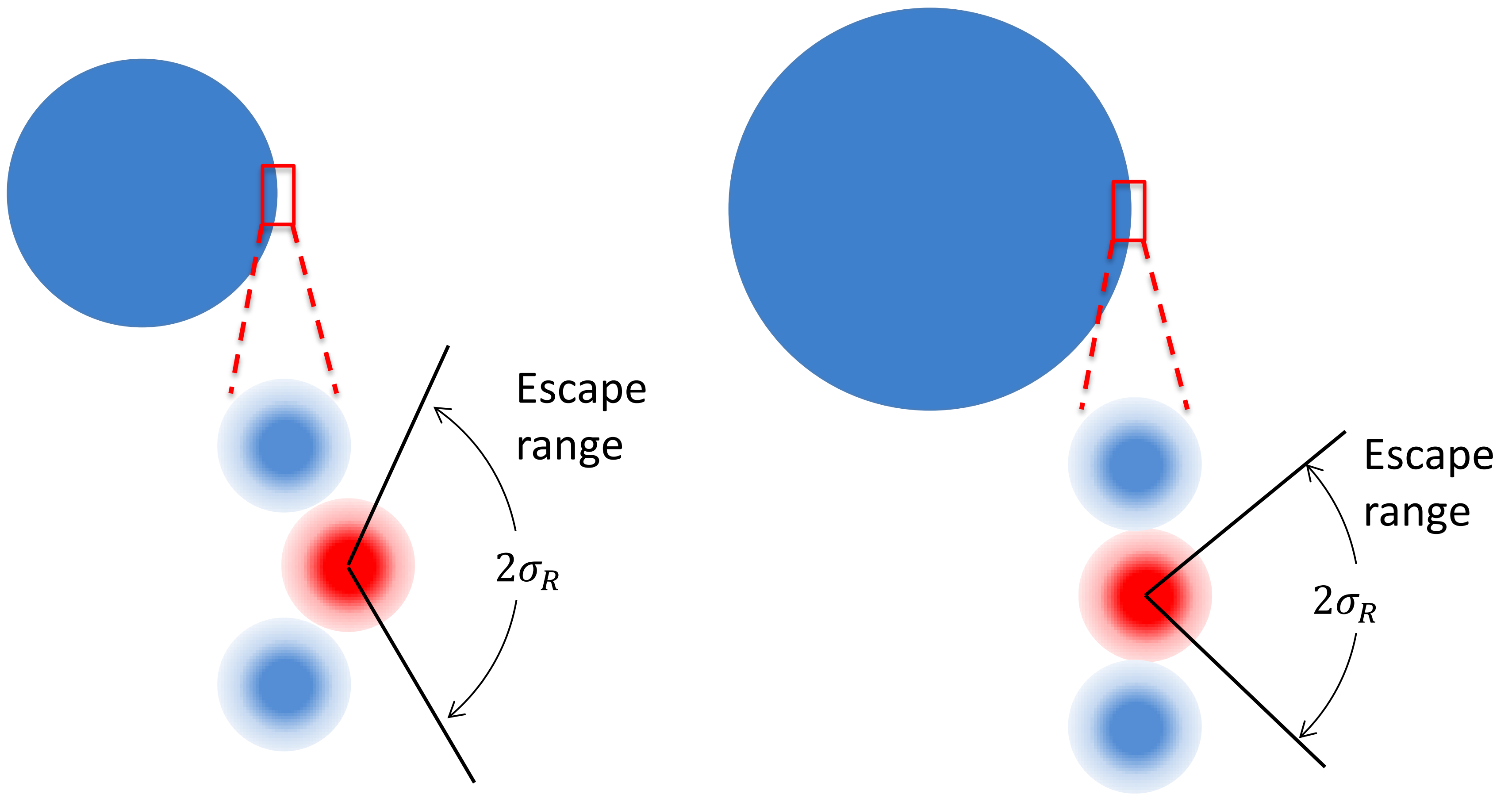
$$f_a > 4a\eta D_r \left(\frac{\rho_c}{\rho_v}\right)$$

Lee, *arXiv:1503.08674*



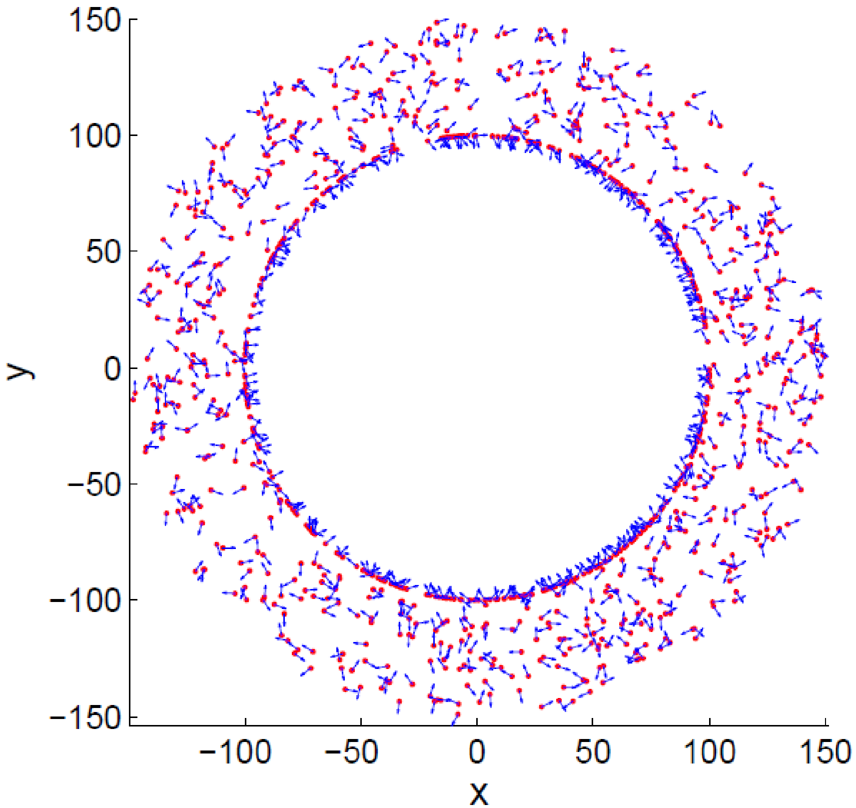
Gibbs-Thomson relation in MIPS

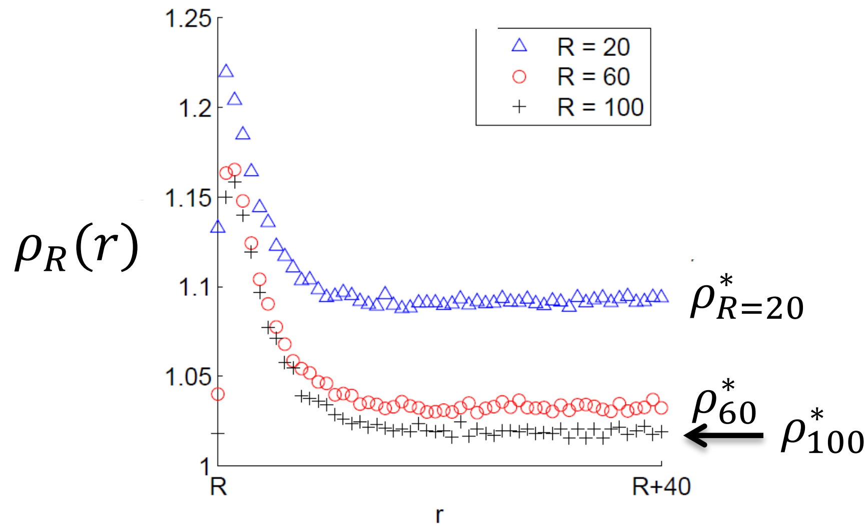
Circular drops: Caging effects

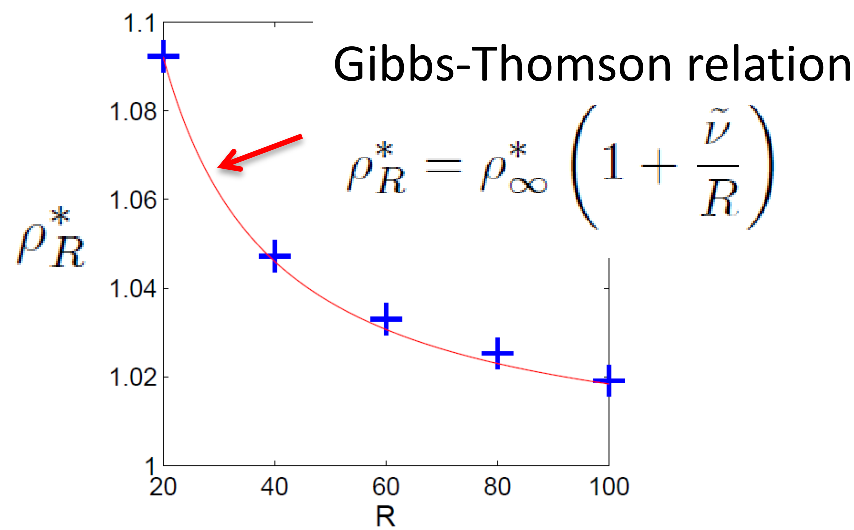


Caging effect of neighbouring particles leads a curvature-dependent escape range: $\sigma_R = A + \frac{B}{R} + \mathcal{O}(R^{-2})$

Circular drop: numerics







Lee, *arXiv:1503.08674*

Universality of coarsening in MIPS

1. Active particles move diffusively across long distance

2. Gibbs-Thomson relation at the interface MIPS droplets

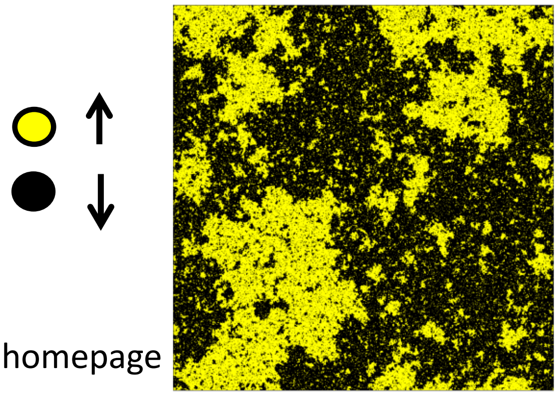
1. + 2. lead to Lifshitz-Slyozov (LS) equilibrium coarsening:

- $\langle R(t) \rangle \sim t^{1/3}$
- Universal (LS) droplet size distribution


Universality

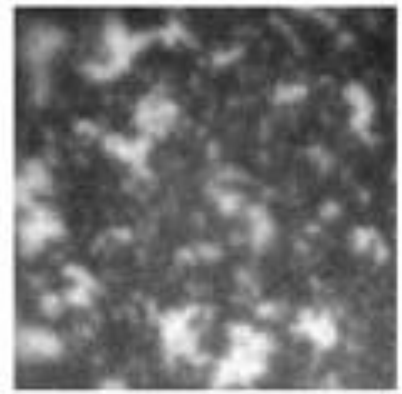
Universality in physics

Ferromagnetism with an easy axis at criticality



P. Colon's homepage

Phase separation in multi-component lipid membrane at criticality



Honerkamp-Smith et al. (2009) BBA

As the measurement is at a larger and larger length scale ($l \to \infty$)

Ising model: $E = -J \sum_{<i.j>} s_i \, s_j$

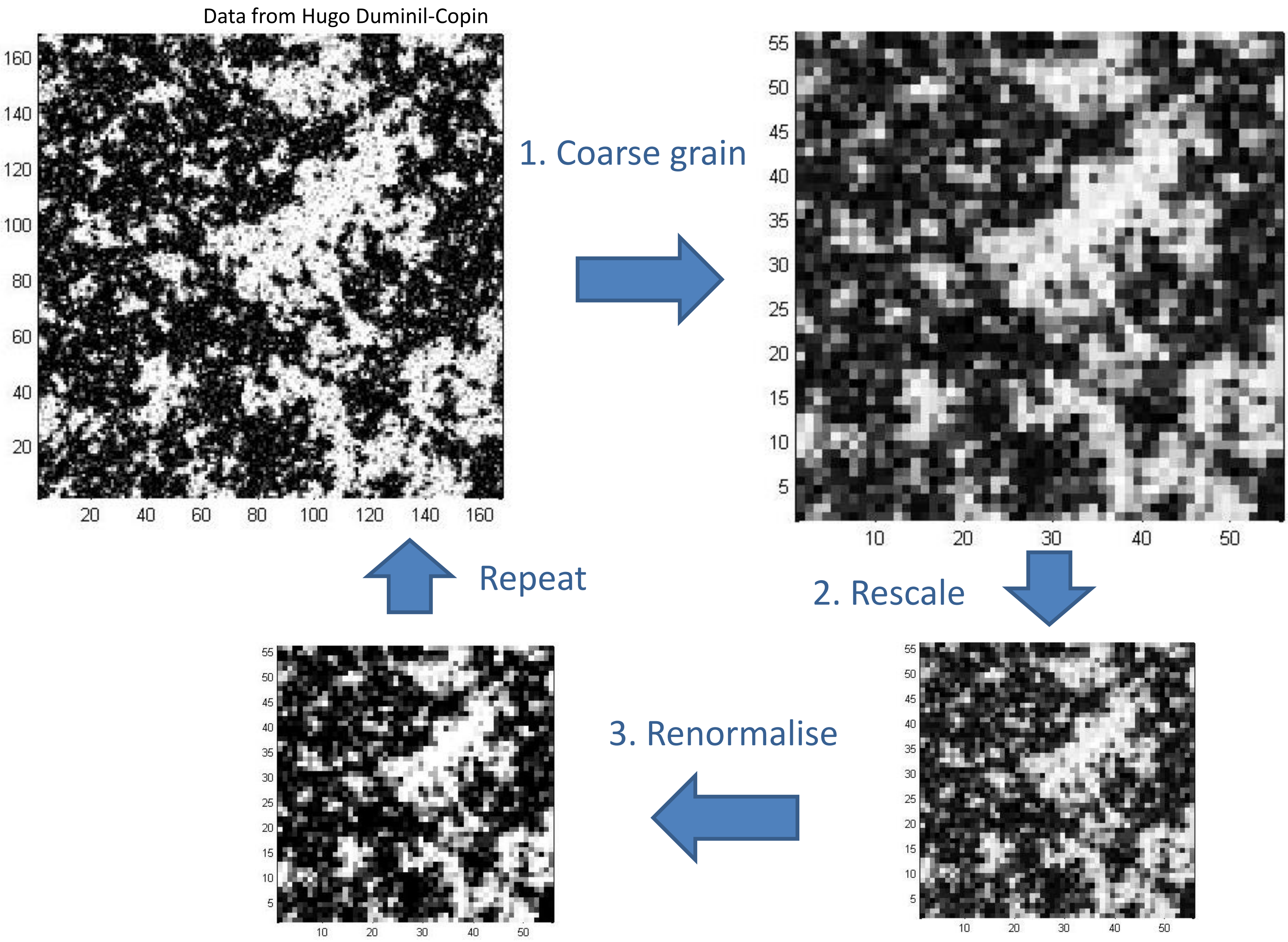
Proved by renormalisation group methods

Kenneth Wilson, Nobel prize 1982

Renormalisation group transformation

$$H_0 = \int d^d r \Big[(\nabla S(\mathbf{r}))^2 + t_0 S(\mathbf{r})^2 + u_0 S(\mathbf{r})^4 + g_6 S(\mathbf{r})^6 + \lambda_4 (\nabla S(\mathbf{r}))^4 + \dots \Big]$$

1. Coarse grain out short wavelength ($k > l^{-1}$) fluctuations
2. Rescale
3. Renormalise


Data from Hugo Duminil-Copin
1. Coarse grain
2. Rescale
3. Renormalise
Repeat

Renormalisation group transformation

$$H_0 = \int d^d r \left[(\nabla S(\mathbf{r}))^2 + t_0 S(\mathbf{r})^2 + u_0 S(\mathbf{r})^4 + g_6 S(\mathbf{r})^6 + \lambda_4 (\nabla S(\mathbf{r}))^4 + \ldots \right]$$

1. Coarse grain out short wavelength ($k > l^{-1}$) fluctuations
2. Rescale
3. Renormalise

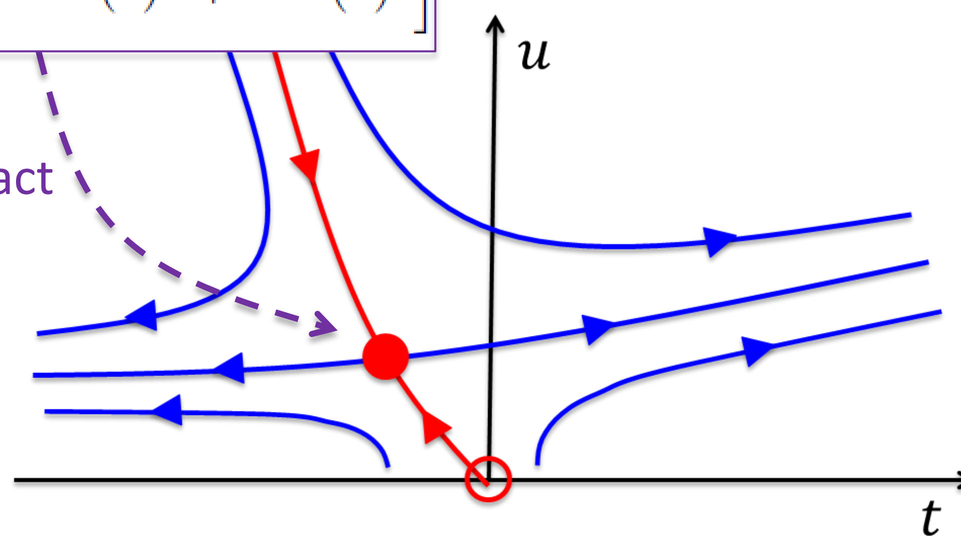
$$H_l = \int d^d r \left[(\nabla S(\mathbf{r}))^2 + t_l S(\mathbf{r})^2 + u_l S(\mathbf{r})^4 \right]$$

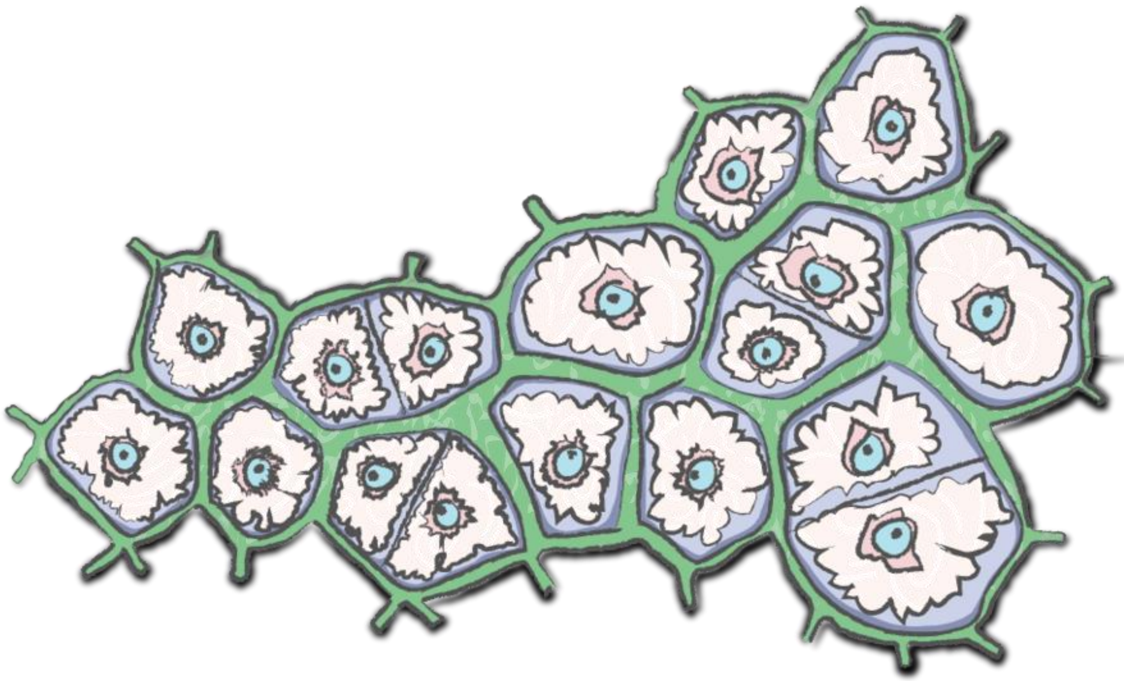
l goes up further

$$\boxed{H^* = \int d^d r \left[(\nabla S(\mathbf{r}))^2 + t^* S(\mathbf{r})^2 + u^* S(\mathbf{r})^4 \right]}$$

Toy model H^* becomes the (asymptotically) exact model

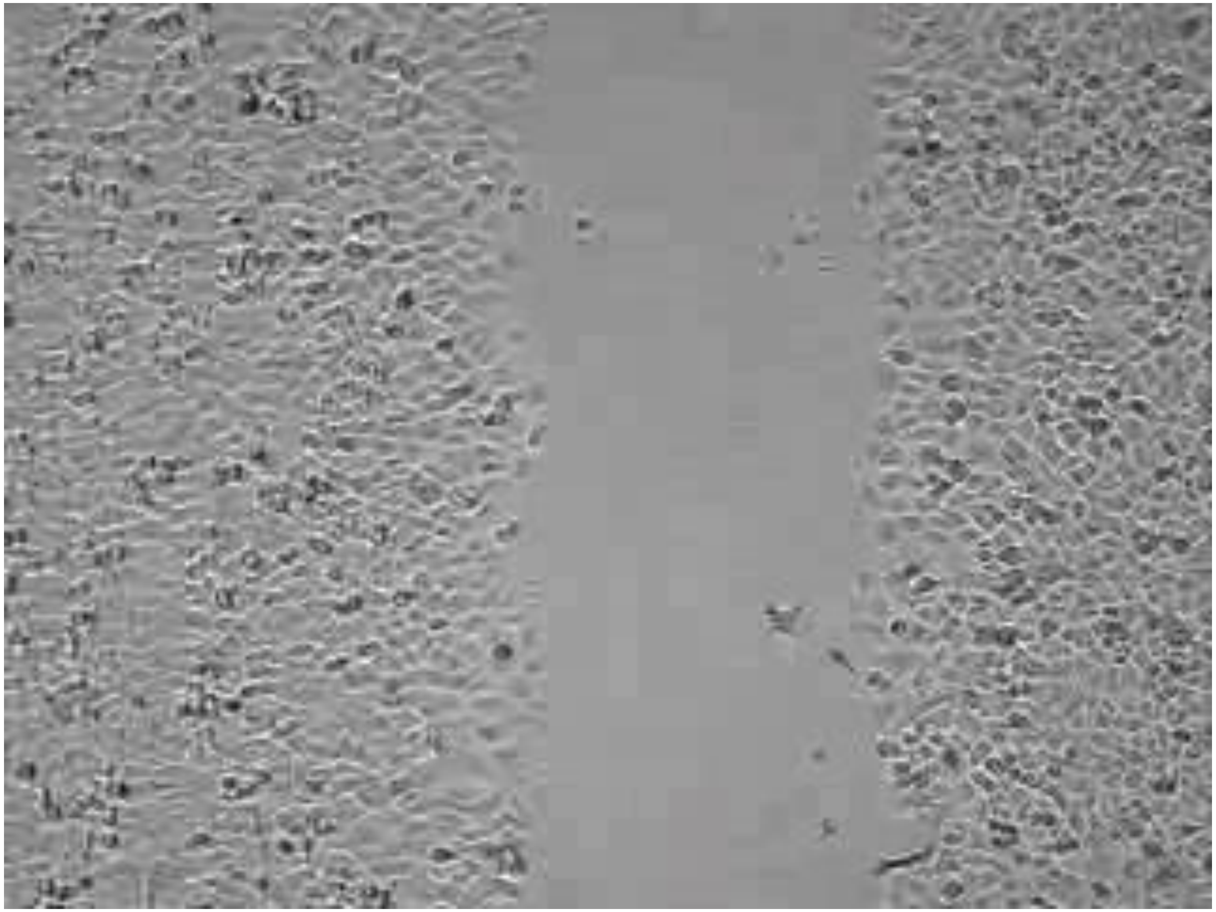
Level of coarse-graining





Incompressible polar active fluids

Wound healing assay



https://www.youtube.com/watch?v=v9xq_GiRXeE

Navier-Stokes description?

$$\partial_t \vec{v} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} = \mu \nabla^2 \vec{v} - \rho^{-1} \vec{\nabla} P$$

Not enough!

Navier-Stokes description?

$$\partial_t \vec{v} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} = (a + bv^2)\vec{v} + \mu \nabla^2 \vec{v} - \rho^{-1} \vec{\nabla} P$$

Not enough!

Missing elements:

- Cells are motile -> Need *active* terms like $(a + bv^2)\vec{v}$

Navier-Stokes description?

$$\partial_t \vec{v} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} = (a + bv^2)\vec{v} + \mu \nabla^2 \vec{v} - \rho^{-1} \vec{\nabla} P + \vec{f}$$

Not enough!

Missing elements:

- Cells are motile -> Need *active* terms like $(a + bv^2)\vec{v}$
- Fluctuations can be important -> Gaussian noise $\vec{f}$

Navier-Stokes description?

$$\partial_t \vec{v} + \lambda(\vec{v} \cdot \vec{\nabla})\vec{v} = (a + bv^2)\vec{v} + \mu \nabla^2 \vec{v} - \rho^{-1} \vec{\nabla} P + \vec{f}$$

Not enough!

Missing elements:

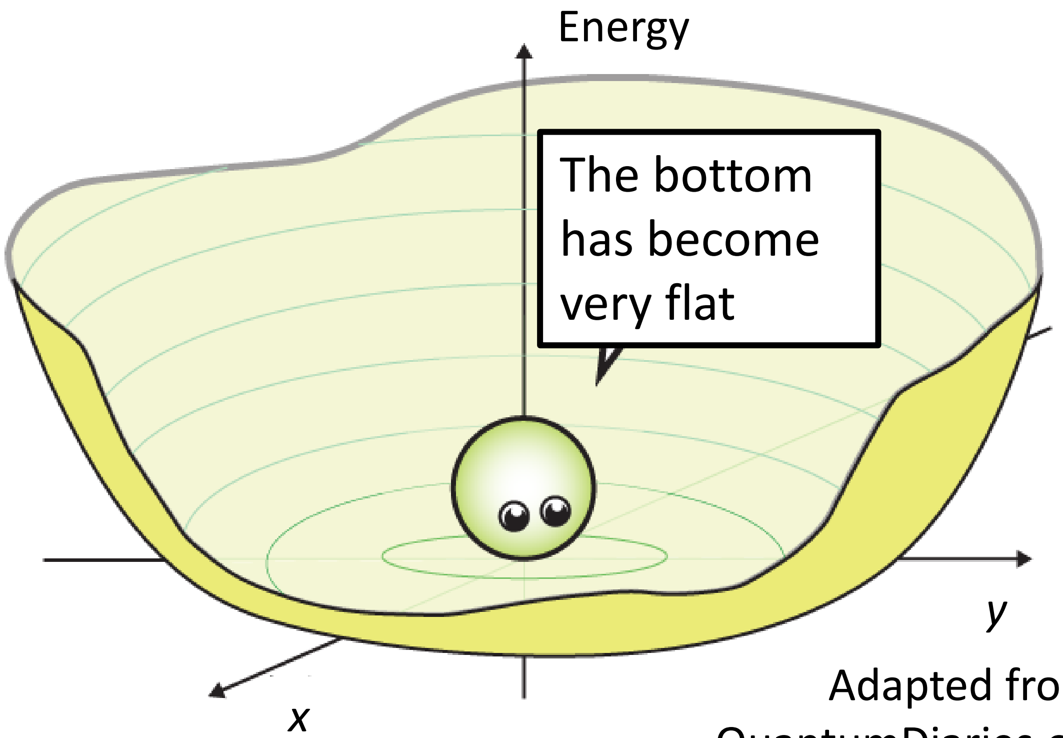
- Cells are motile -> Need *active* terms like $(a + bv^2)\vec{v}$
- Fluctuations can be important -> Gaussian noise $\vec{f}$
- No Galilean invariance -> prefactor of $(\vec{v} \cdot \vec{\nabla})\vec{v}$ may not be 1

In fact, to be completely general…

There can be infinitely many more red terms

$$\partial_t \vec{v} + \rho^{-1}\vec{\nabla}P - \vec{f} = -\lambda\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} - (a + bv^2)\vec{v} - \mu\nabla^2\vec{v}$$
$$+cv^4\vec{v} + \xi(\nabla^2)^2\vec{v} + \cdots$$

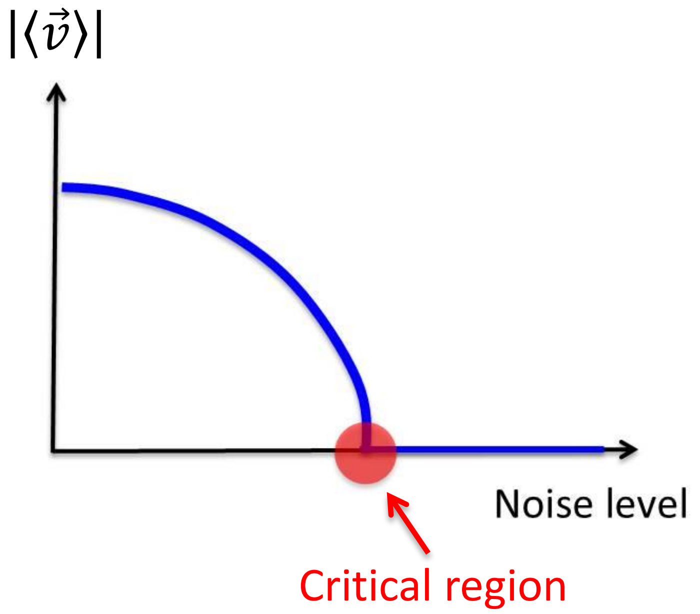
With so many parameters, can we ever say something universal about the system?



Adapted from QuantumDiaries.org

Incompressible active fluids at criticality

Phase behaviour of incompressible active fluids



Critical incompressible polar active fluids

EOM: $\partial_t \vec{v} + \vec{\nabla} P - \vec{f} = -\lambda(\vec{v}\cdot\vec{\nabla})\vec{v} - (a + bv^2)\vec{v} - \mu\nabla^2\vec{v} + cv^4\vec{v} + \xi(\nabla^2)^2\vec{v} + \cdots$

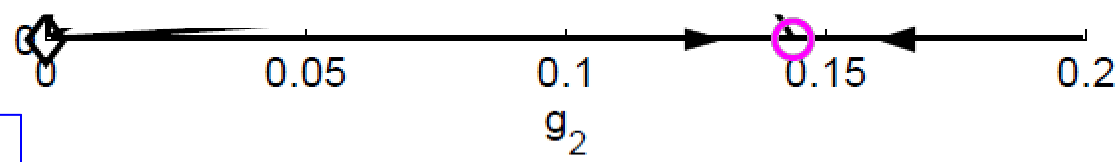
RG transformation

$$\partial_t \vec{v} + \vec{\nabla} P + \overrightarrow{f_l} = -\lambda_l(\vec{v}\cdot\vec{\nabla})\vec{v} - (a_l + b_l v^2)\vec{v} - \mu_l\nabla^2\vec{v}$$

Exact hydrodynamic EOM with **TWO** coefficients governing the model's scale-invariance properties:

$$g_1(l) \sim \frac{D_l {\lambda_l}^2}{{\mu_l}^3} \quad , \quad g_2(l) \sim \frac{D_l b_l}{{\mu_l}^2}$$

Level of coarse-graining

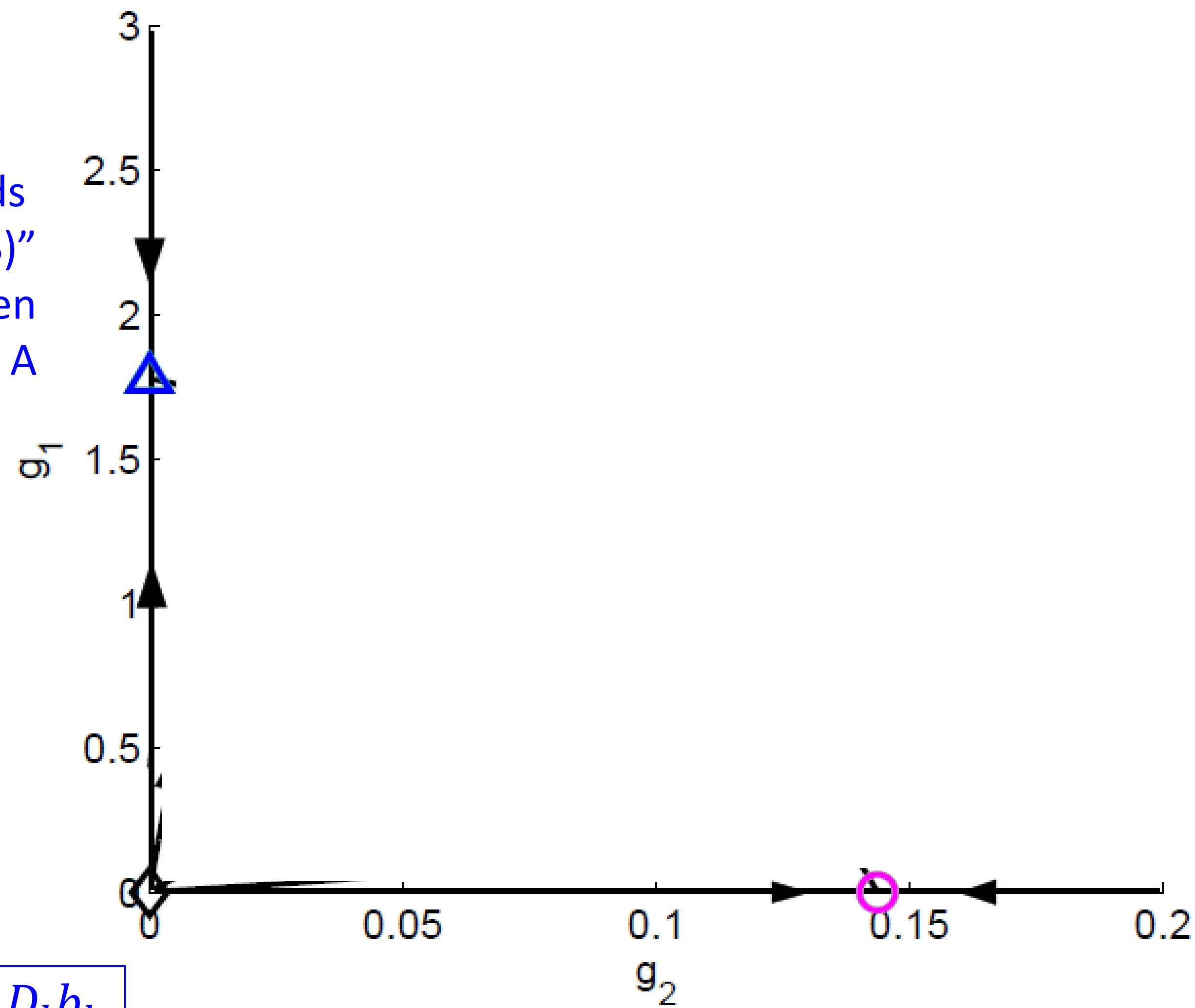


$$g_1(l) \sim \frac{D_l \lambda_l^{\,2}}{\mu_l^{\,3}}; \; g_2(l) \sim \frac{D_l b_l}{\mu_l^{\,2}}$$

"Ferromagnets with dipolar interactions"
Aharony and Fisher (1973) Phys. Rev. Lett.

Chen, Toner, Lee (2015)
New J. Phys. **17**, 042002

“Randomly stirred fluids (Model B)”
Forster, Nelson & Stephen (1977) Phys. Rev. A



$$g_1(l) \sim \frac{D_l \lambda_l{}^2}{\mu_l{}^3};\; g_2(l) \sim \frac{D_l b_l}{\mu_l{}^2}$$

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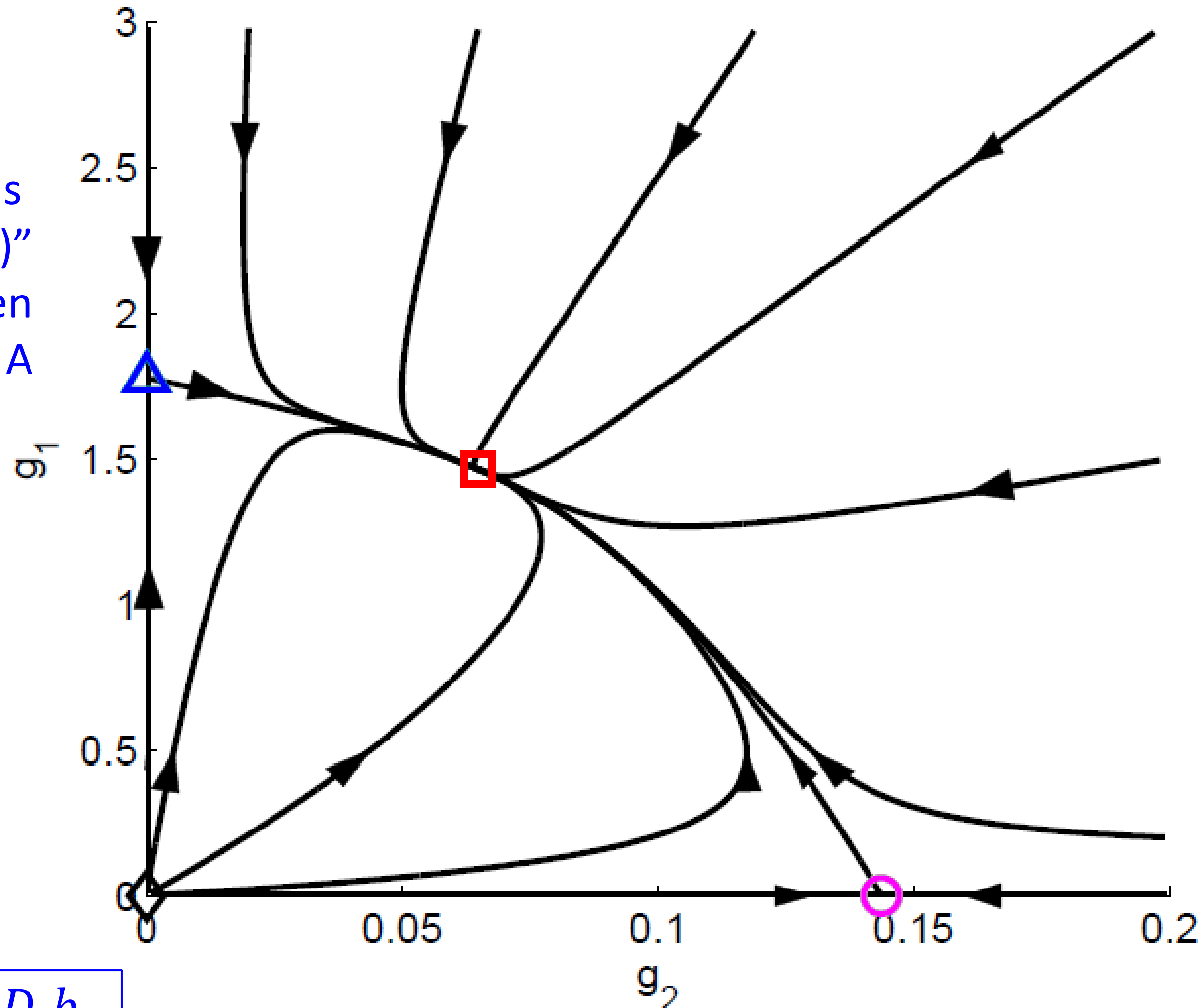

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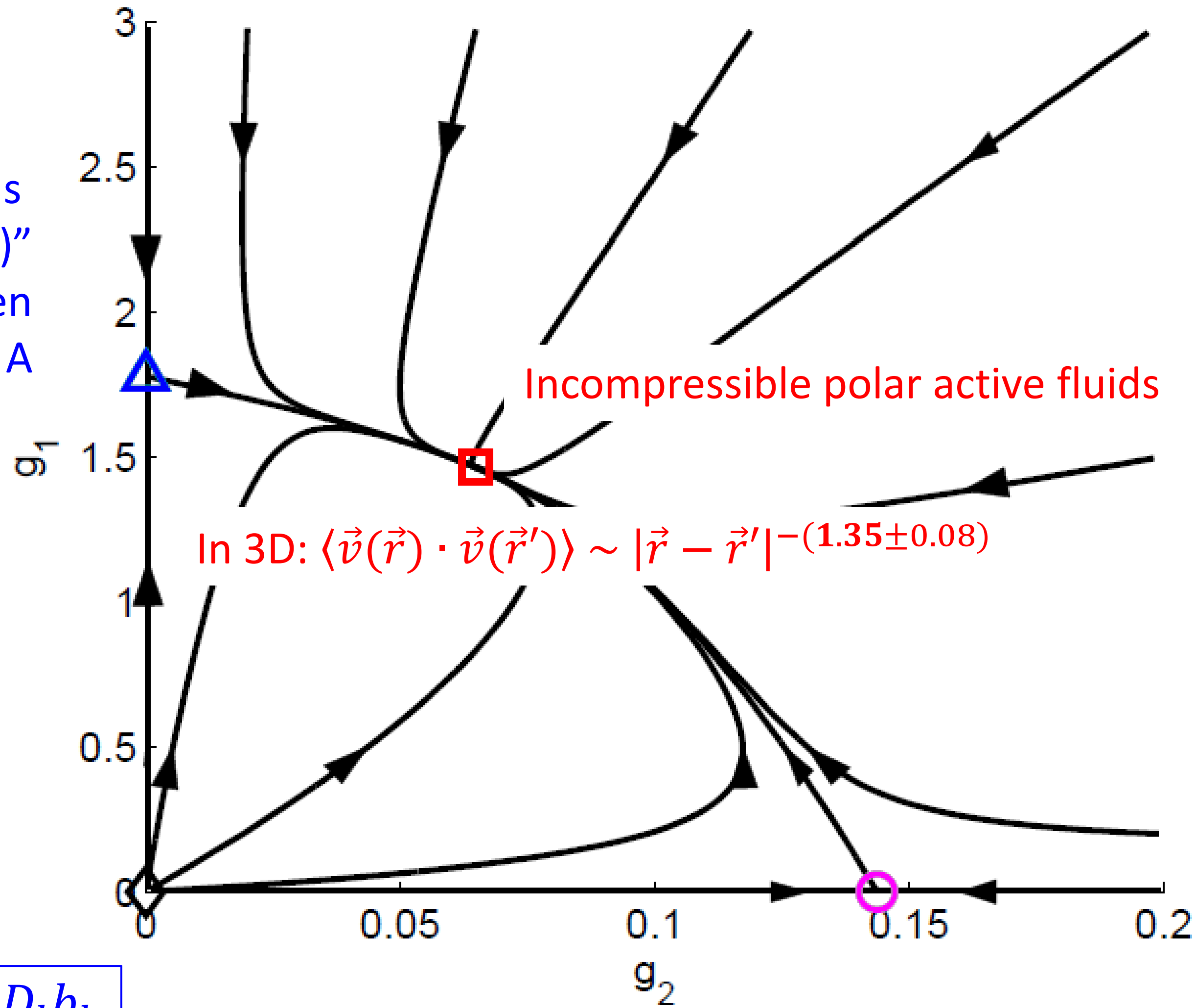

$$g_1(l) \sim \frac{D_l {\lambda_l}^2}{{\mu_l}^3}; \; g_2(l) \sim \frac{D_l b_l}{{\mu_l}^2}$$

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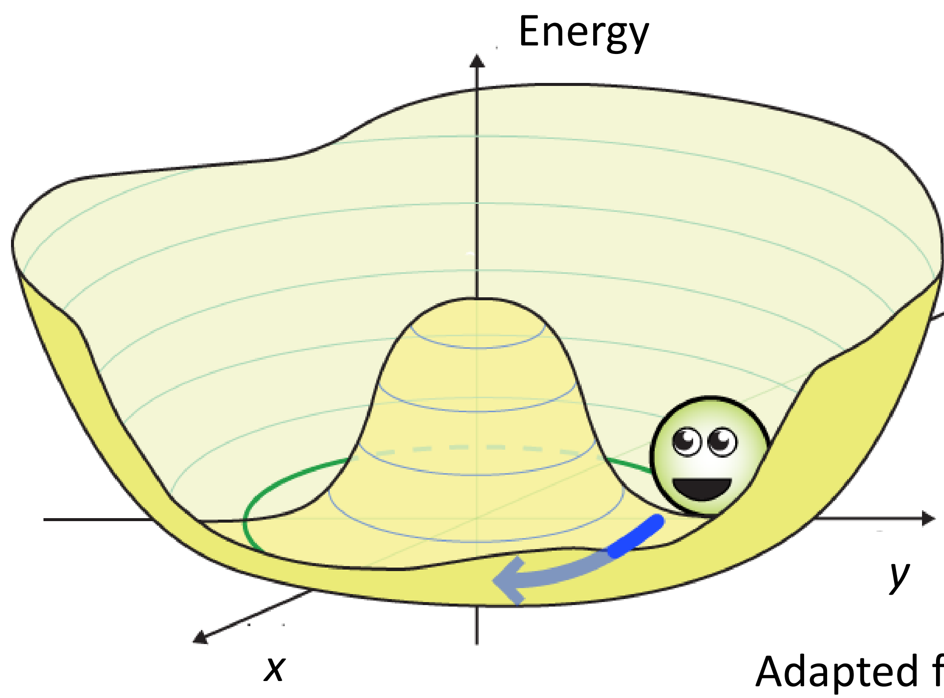
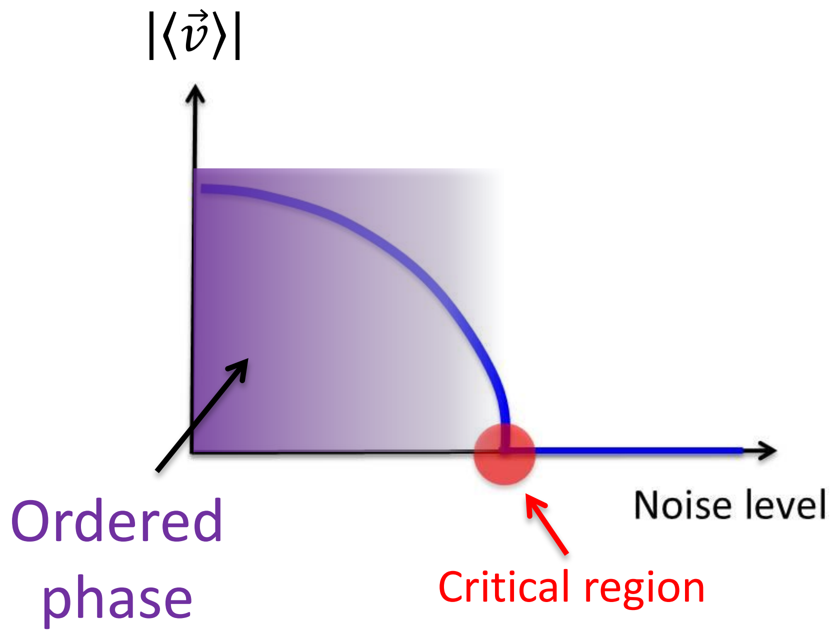


Adapted from
QuantumDiaries.org

Incompressible active fluids in the ordered phase

Ordered incompressible polar active fluids



- **Universality is more than criticality!**
- Universal behaviour expected in the symmetry-broken phase of a continuous symmetry

Belitz, Kirkpatrick, Vojta (2005) Rev. Mod. Phys.

Ordered phase of 2D incompressible active fluids

EOM:

$$\partial_t \mathbf{v} + \lambda(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla P + (a - bv^2)\mathbf{v} + \mu\nabla^2\mathbf{v} + \mathbf{f} + \dots$$

1. RG transformation

$$\begin{aligned}\partial_t u_x &= -\partial_x P - 2b\left(u_x + \frac{u_y^2}{2v_0}\right) + \mu\nabla^2 u_x + f_x \\ \partial_t u_y &= -\partial_y P - \frac{2b}{v_0}\left(u_x + \frac{u_y^2}{2v_0}\right)u_y + \mu\nabla^2 u_y + f_y\end{aligned}$$

Ordered phase: $\mathbf{v} = (v_0 + u_x(\mathbf{r},t))\hat{x} + u_y(\mathbf{r},t)\hat{y}$

2. Use streaming function h to enforce Incompressibility:

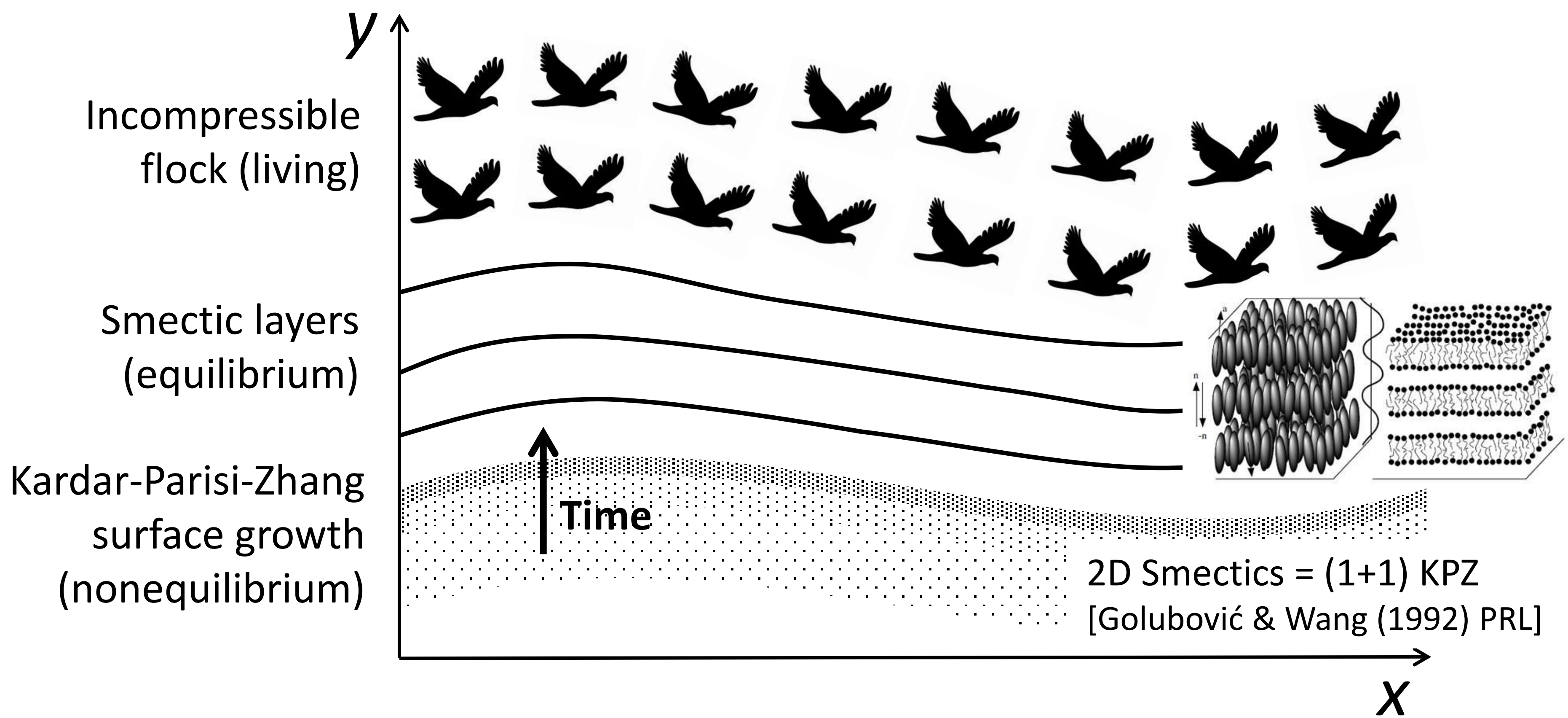
$$\begin{aligned}u_x &= -v_0\partial_y h \\ u_y &= v_0\partial_x h\end{aligned}$$

3. Equal-time (static) correlations in 2D active fluids are mapped onto the 2D smectic model:

$$H_{\text{smectic}} = \int d^2r \left[bv_0\left(\partial_y h - \frac{(\partial_x h)^2}{2}\right)^2 + \mu v_0^2\left(\partial_x^2 h\right)^2 \right]$$

Chen, Lee, Toner, arXiv:1601.01924

Universality of Kardar-Parisi-Zhang



Chen, Lee, Toner,
arXiv:1601.01924

KPZ model: $\partial_t h = \nu \nabla^2 h + \lambda (\nabla h)^2 + \eta$

Suppression of the coffee-ring effect by shape-dependent capillary interactions

Peter J. Yunker[1], Tim Still[1,2], Matthew A. Lohr[1] & A. G. Yodh[1]

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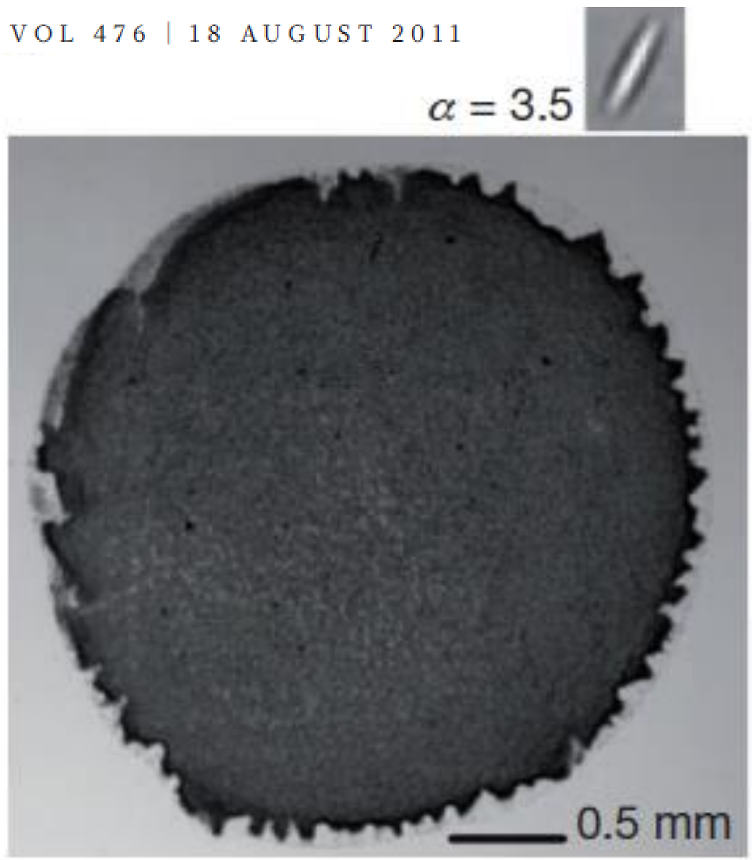
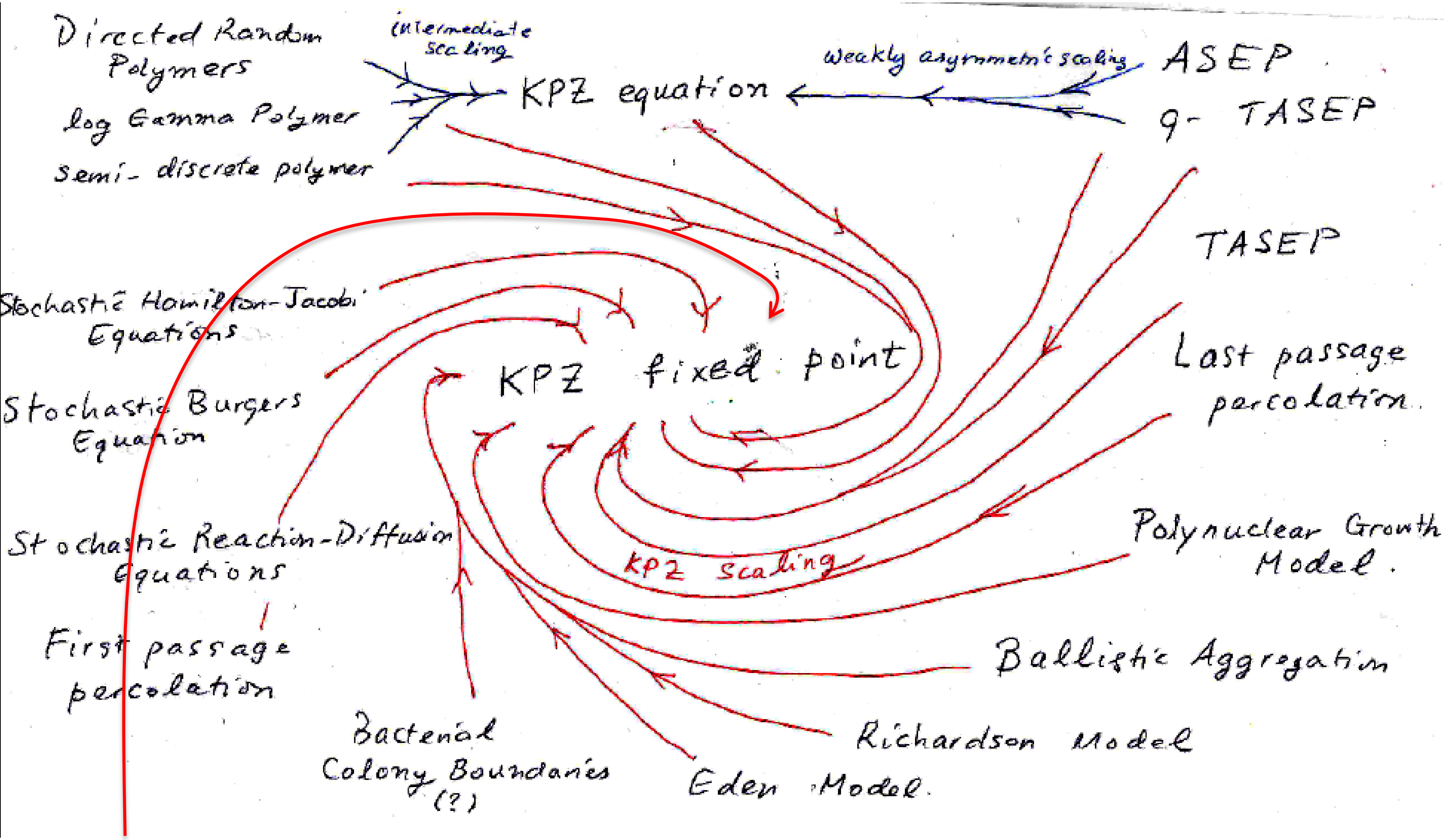


Slide from J. Quastel (Toronto)



2D incompressible active fluids in the ordered phase

[Chen, Lee & Toner (2016) arXiv:1601.01924]

M.C. Escher (1938) *Day and Night*

Outlook

‘Periodic table’ of universality classes

Equilibrium

Designation	System
A	Kinetic Ising anisotropic magnets
B	Kinetic Ising uniaxial ferromagnet
C	Anisotropic magnets structural transition
H	Gas–liquid binary fluid
E	Easy-plane magnet, $h_z = 0$
F	Easy-plane magnet, $h_z \neq 0$ superfluid helium
G	Heisenberg antiferromagnet
J	Heisenberg ferromagnet

Hohenberg and Halperin (1977) “Theory of dynamic critical phenomena” Rev. Mod. Phys.

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Non-equilibrium

Randomly stirred fluids
[Forster, Nelson & Stephen (1977) Phys. Rev. A]

Directed percolation
[Kinzel (1983)]

Kardar-Parisi-Zhang
[KPZ (1983) Phys. Rev. Lett.]

Living matter

Incompressible active polar fluids
[Chen, Toner, Lee (2015) New J. Phys.]

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Non-equilibrium

Transition	Ordered phase
?	?
?	?
?	?
?	LSW
?	KPZ
?	?
?	?
New Universality class	?

Universalities in Biology

	Biology		
Designation	Biological System	Transition	Ordered phase
A	?	?	?
B	?	?	?
C	?	?	?
H	Motile bacteria	?	LSW
E	Epithelial tissue	?	KPZ
F	?	?	?
G	?	?	?
J	Birds	New Universality class	?

Summary

- Universality is everywhere in biology
- Universal coarsening kinetics (LSW) in motility-induced phase separation [Lee, arXiv:1503.08674]
- Universal behaviour of incompressible polar active fluids
  - At criticality (New universality class) [Chen, Toner, Lee (2015) New J. Phys.]
  - Ordered phase in 2D (KPZ) [Chen, Lee, Toner, arXiv:1601:01924]
- Future direction: Categorisation of universality of living matter

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John Toner (*Oregon*)

EPSRC
Engineering and Physical Sciences Research Council

BBSRC
bioscience for the future

Thank you for your Attention!