

LPTMC, Sorbonne University & CNRS

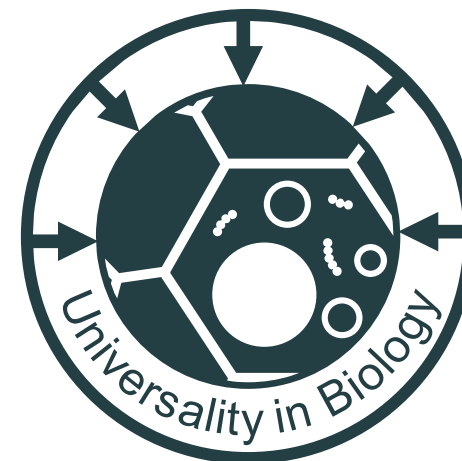
15 February 2024

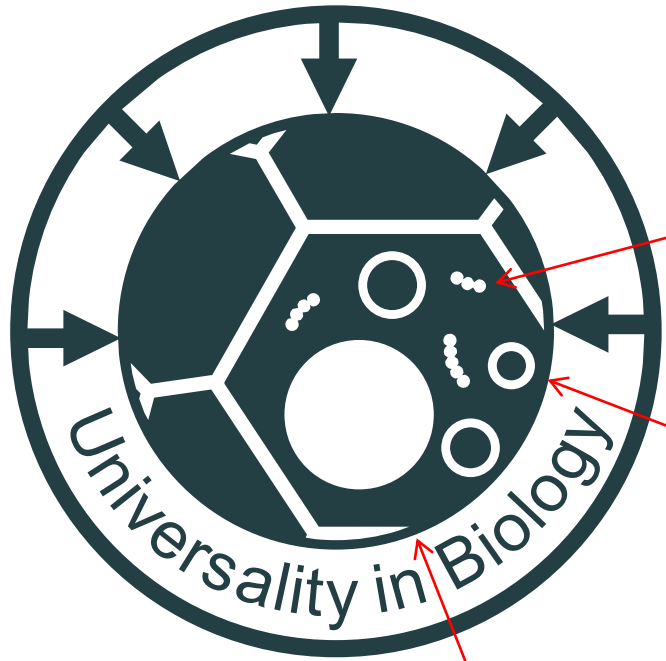
A new universality class describes Vicsek's flocking phase in physical dimensions

Patrick Jentsch & Chiu Fan Lee

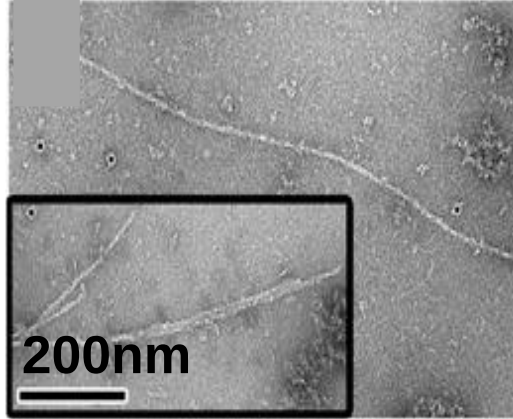
Department of Bioengineering, Imperial College London, UK

Imperial College
London



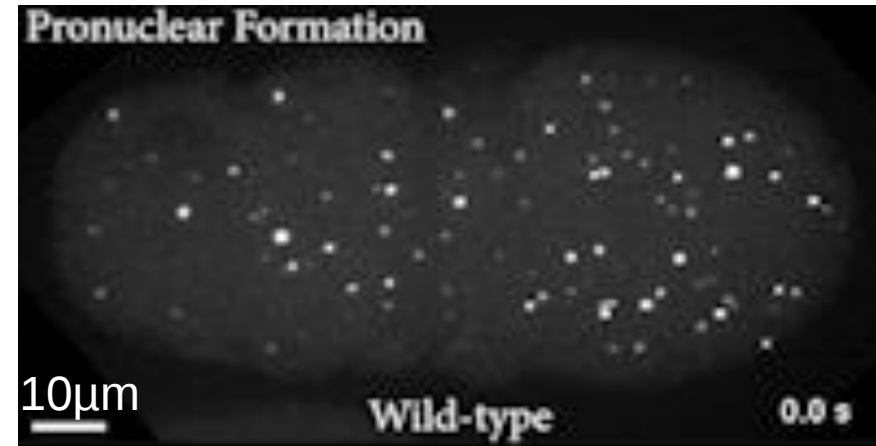


Amyloid formation



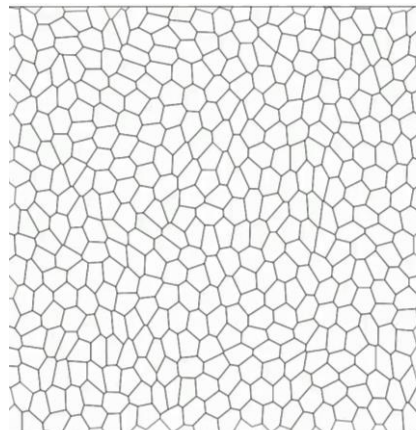
Pytowski, Lee, Foley, Vaux, Jean (2020)
Liquid-liquid phase separation of type II diabetes associated IAPP initiates hydrogelation and aggregation
 PNAS

Biomolecular condensation



Folkmann, Putnam, Lee, Seydoux (2021)
Regulation of biomolecular condensates by interfacial protein clusters
 Science

Active matter



Killeen, Bertrand, Lee (2022)
Polar Fluctuations Lead to Extensile Nematic Behavior in Confluent Tissues
 Phys Rev Lett

Acknowledgement



**John Toner
(Oregon)**



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(China U
Mining &
Technology)**

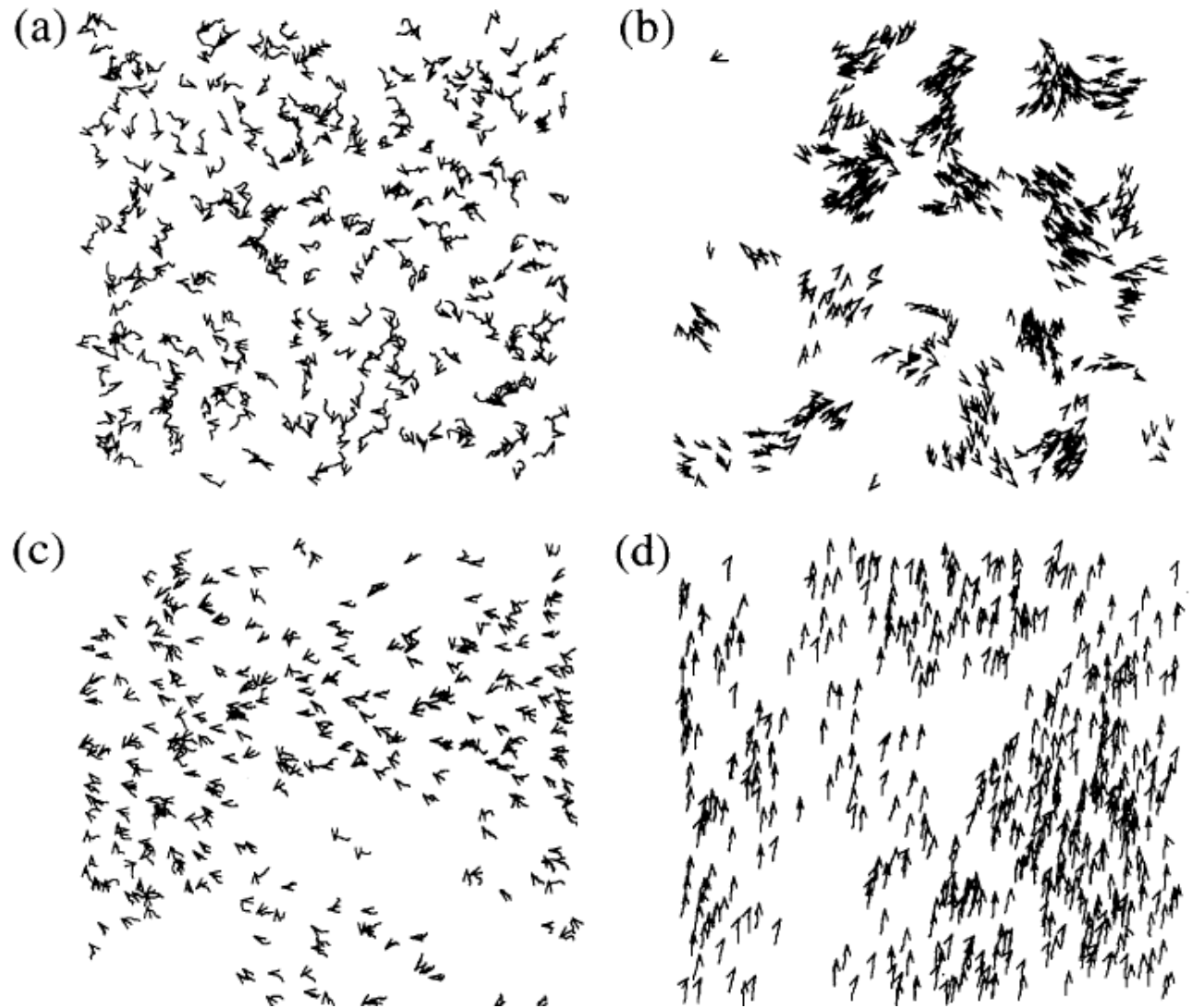


**Ananyo
Maitra
(CNRS, CY
Cergy Paris
Université)**

Plan of talk

1. A brief & biased history of active matter physics
2. Simplified Toner-Tu model
3. Linear theory
4. Nonlinear theory using FRG
5. FRG fixed points
6. Analytical treatment
7. Summary & Outlook

1. A brief & biased history of active matter physics



A simulation model for flocking in 1995

VOLUME 75, NUMBER 6

PHYSICAL REVIEW LETTERS

7 AUGUST 1995

Novel Type of Phase Transition in a System of Self-Driven Particles

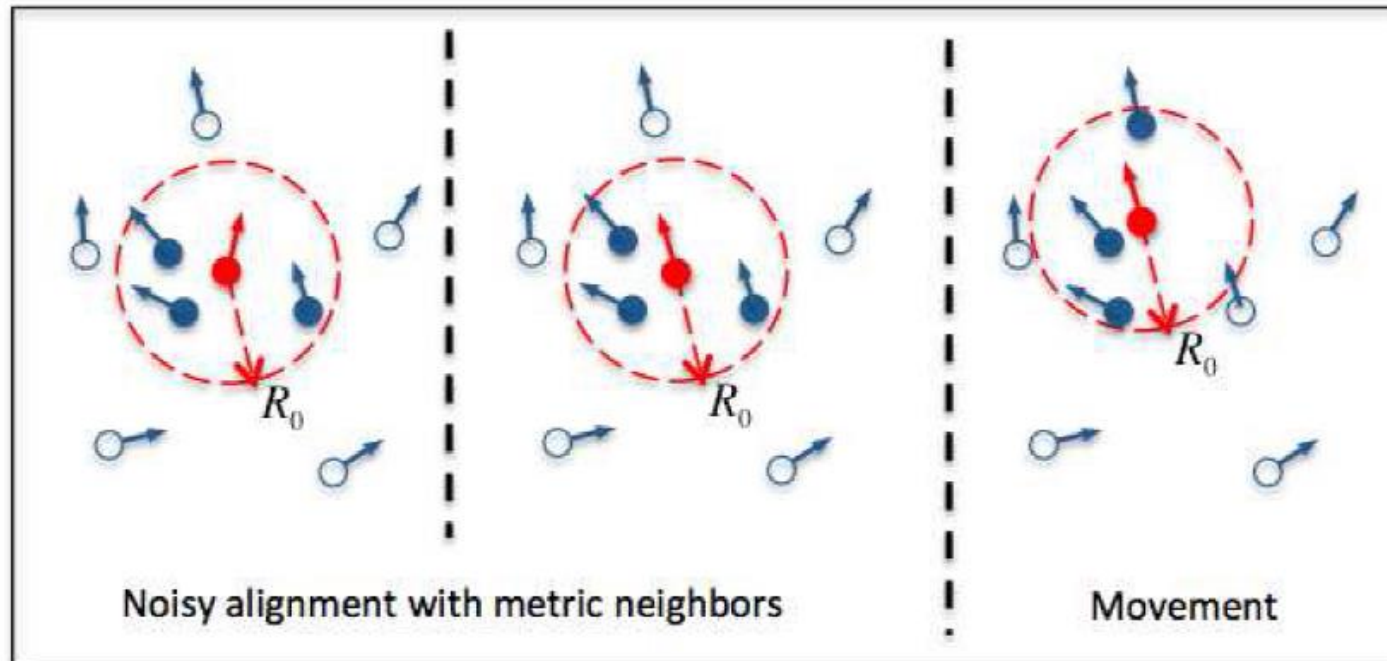
Tamás Vicsek,^{1,2} András Czirók,¹ Eshel Ben-Jacob,³ Inon Cohen,³ and Ofer Shochet³

¹Department of Atomic Physics, Eötvös University, Budapest, Puskin u 5-7, 1088 Hungary

²Institute for Technical Physics, Budapest, P.O.B. 76, 1325 Hungary

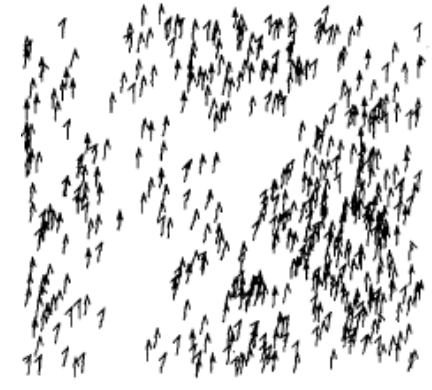
³School of Physics, Tel-Aviv University, 69978 Tel-Aviv, Israel

(Received 25 April 1994)



Homogeneous
disordered (D)

Homogeneous
ordered (O)

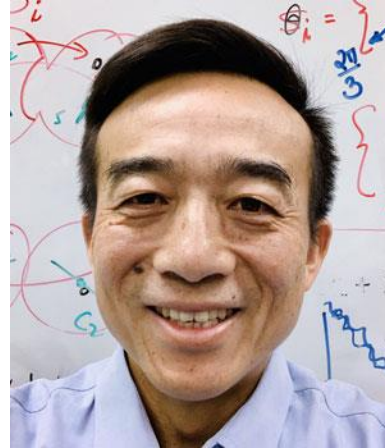
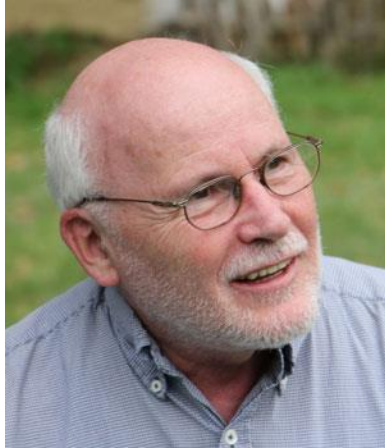


Noise level

What's the big deal?

- A continuous symmetry (rotational symmetry) seems to be broken in 2D!
 - A violation of the Mermin-Wagner-Hohenberg theorem at thermal equilibrium
 - Non-equilibrium dynamics is a crucial ingredient!
- “Order-disorder” may be a new type of transition [Vicsek et al (1995) PRL]
- Flocking phase may be a nonequilibrium phase [Toner & Tu (1995) PRL]

2020 APS Onsager Prize



Citation: *"For seminal work on the theory of flocking that marked the birth and contributed greatly to the development of the field of active matter."*

See also Tu (2023) *The renormalization group for non-equilibrium systems*, Nat. Phys. **19**, 1536

Intriguingly, controversies soon emerged for both landmark studies !

Controversy 1: Absence of critical transition in the Vicsek model

VOLUME 92, NUMBER 2

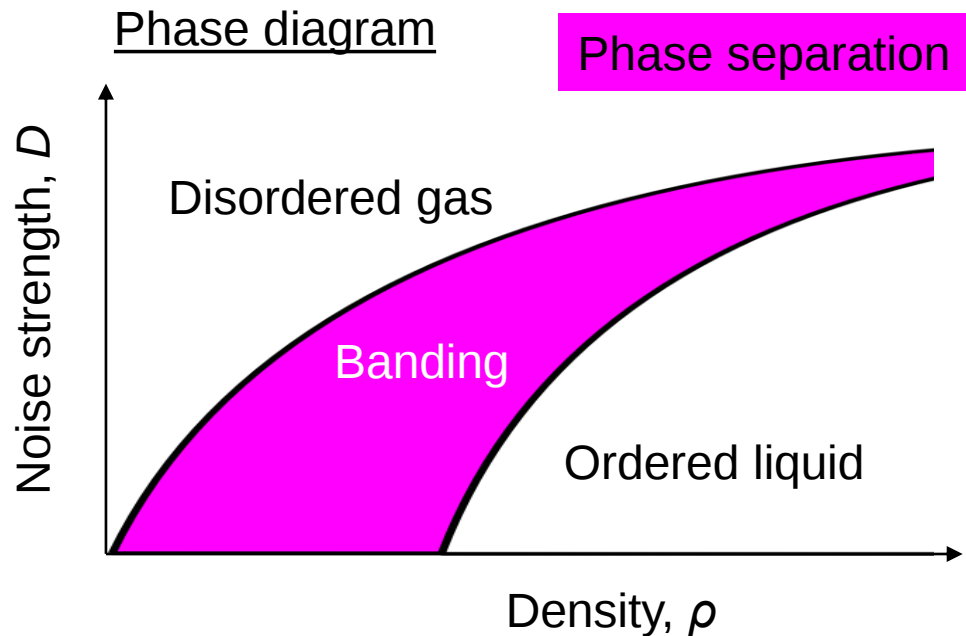
PHYSICAL REVIEW LETTERS

week ending
16 JANUARY 2004

- Critical order-disorder transition is pre-empted by phase separation

Onset of Collective and Cohesive Motion

Guillaume Grégoire and Hugues Chaté



Controversy 1: Absence of critical transition in the Vicsek model

VOLUME 92, NUMBER 2

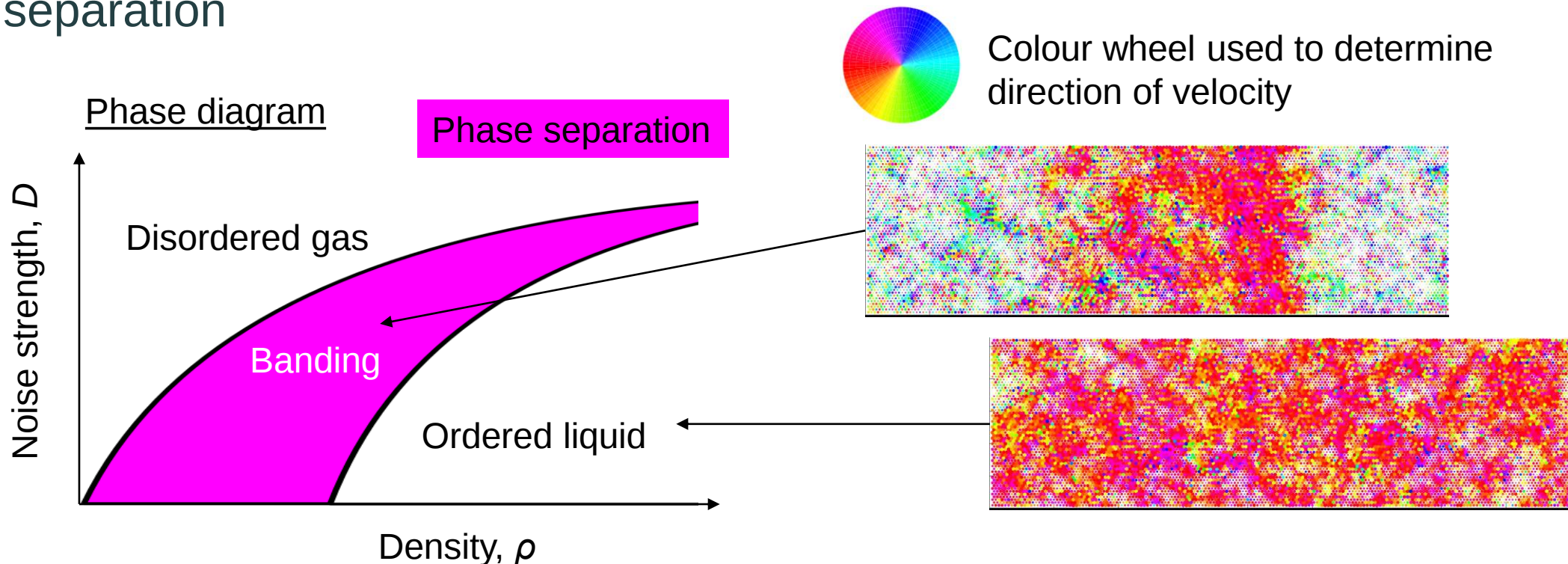
PHYSICAL REVIEW LETTERS

week ending
16 JANUARY 2004

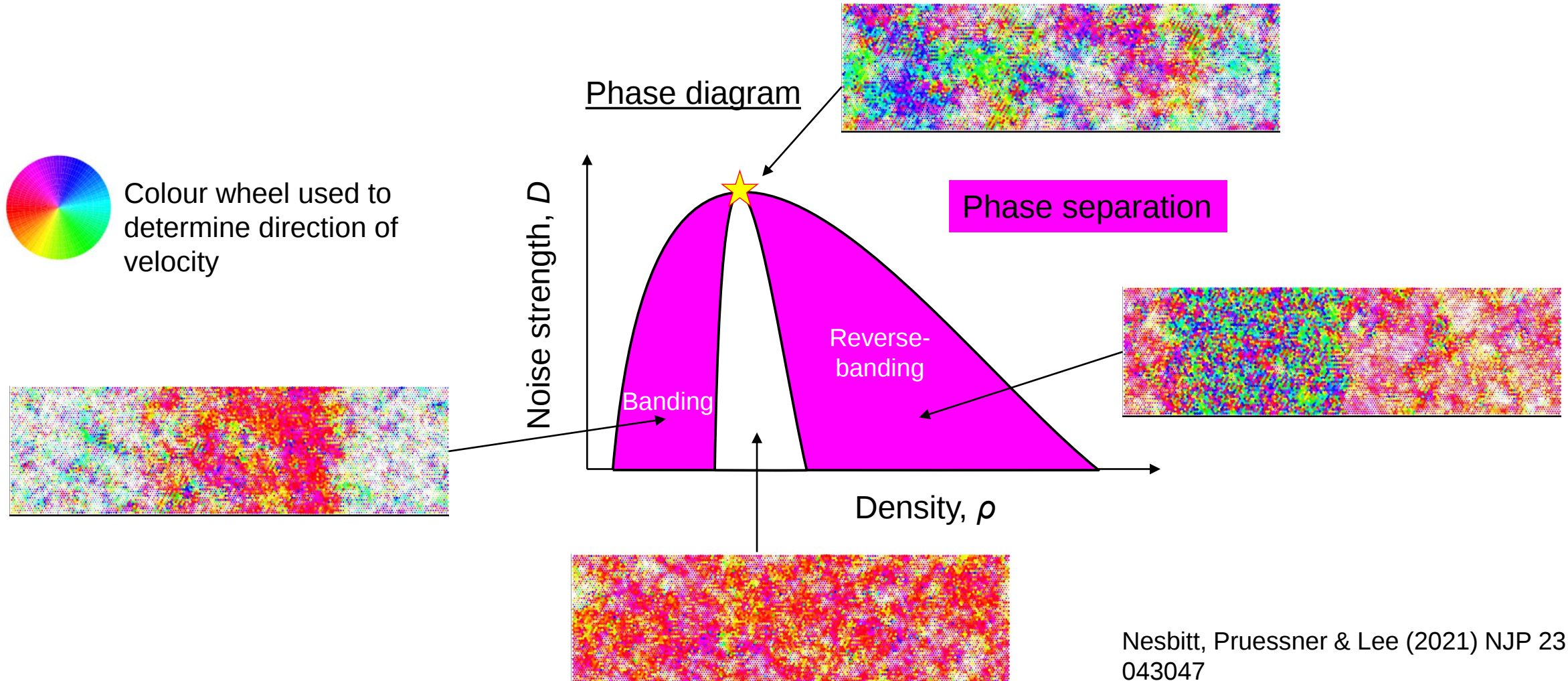
- Critical order-disorder transition is pre-empted by phase separation

Onset of Collective and Cohesive Motion

Guillaume Grégoire and Hugues Chaté



Aside: Critical order-disorder can exist when volume exclusion occurs



Controversy 2: Toner-Tu 95 RG analysis not as general as originally thought

PHYSICAL REVIEW E **86**, 031918 (2012)

- Missing terms leading to potentially invalid scaling exponents

Reanalysis of the hydrodynamic theory of fluid, polar-ordered flocks

John Toner

PHYSICAL REVIEW LETTERS **123**, 218001 (2019)

Editors' Suggestion

Quantitative Assessment of the Toner and Tu Theory of Polar Flocks

Benoît Mahault^{1,2}, Francesco Ginelli^{3,4} and Hugues Chaté^{1,5,6}

Spatial dimension (d)	χ	z	ζ
$d = 2$:			
TT 95	-0.20	1.20	0.6
simulation	-0.31(2)	1.33(2)	0.95(2)
$d = 3$:			
TT 95	-0.60	1.60	0.8
simulation	-0.62	1.77	1

← $(t, \mathbf{r}_\perp, x, \delta \mathbf{g}, \delta \rho) \rightarrow (te^{z^l}, \mathbf{r}_\perp e^l, xe^{\zeta^l}, \delta \mathbf{g} e^{\chi^l}, \delta \rho e^{\chi^l})$

Where do we stand?

- No critical behaviour in the Vicsek model
- Universality class (UC) for Vicsek's flocking phase remains elusive

Here, we use nonperturbative, functional RG to study a *simplified* TT model

- TT UC applies for $\frac{11}{3} (\approx 3.67) < d < 4$
- Below $d = 11/3$, a new UC emerges, whose scaling exponents agree remarkably well with simulation in 2D & 3D

Spatial dimension (d)	χ	z	ζ
$d = 2 :$			
TT 95	-0.20	1.20	0.6
simulation	-0.31(2)	1.33(2)	0.95(2)
new UC	-0.325	1.325	0.975
$d = 3 :$			
TT 95	-0.60	1.60	0.8
simulation	-0.62	1.77	1
new UC	-0.65	1.65	0.95



2. Simplified Toner-Tu model

Simplified Toner-Tu model

- Toner-Tu model Toner, Tu, *Phys. Rev. Lett* (1995)
 - Ginzburg-Landau theory for flocking
- *Simplification 1*
 - Constrained fluctuations in flocking direction
→ Reduced linear complexity
- *Simplification 2*
 - No density dependent nonlinearities
→ Simplified nonlinear complexity
- Density fluctuations contribute linearly!

Toner Tu Model

Toner, Tu, *Phys. Rev. Lett* (1995)

Continuity equation for density

$$\partial_t \rho = -\nabla \cdot \mathbf{g}$$

EOM for momentum density

$$\begin{aligned} & \gamma \partial_t \mathbf{g} + \lambda_1 \mathbf{g} \cdot \nabla \mathbf{g} + \lambda_2 \mathbf{g} \nabla \cdot \mathbf{g} + \lambda_3 \nabla(|\mathbf{g}|^2) \\ &= -U(\rho, g^2) \mathbf{g} - P_1(\rho, g^2) \nabla \rho + \mu_1 \nabla^2 \mathbf{g} + \mu_2 \nabla(\nabla \cdot \mathbf{g}) \\ & \quad + \mu_3 (\mathbf{g} \cdot \nabla)^2 \mathbf{g} + P_2 \mathbf{g} (\mathbf{g} \cdot \nabla \rho) + \dots + \mathbf{f} \end{aligned}$$

Under assumptions of:

- Mass conservation
- Translation sym.
- Rotation sym.
- Chiral sym.

Toner Tu Model

Toner, Tu, *Phys. Rev. Lett* (1995)

Continuity equation for **density**

$$\partial_t \rho = -\nabla \cdot \mathbf{g}$$

Simplification 1:

No fluctuations in
flocking direction

$$\delta \mathbf{g}_x = 0$$

Simplification 2:

No nonlinear
density terms

EOM for **momentum** density

$$\begin{aligned} & \gamma \partial_t \mathbf{g} + \lambda_1 \mathbf{g} \cdot \nabla \mathbf{g} + \lambda_2 \mathbf{g} \nabla \cdot \mathbf{g} + \lambda_3 \nabla(|\mathbf{g}|^2) \\ &= -U(\cancel{\rho} g^2) \mathbf{g} - P_1(\cancel{\rho} g^2) \nabla \rho + \mu_1 \nabla^2 \mathbf{g} + \mu_2 \nabla(\nabla \cdot \mathbf{g}) \\ & \quad + \mu_3 (\mathbf{g} \cdot \nabla)^2 \mathbf{g} + \cancel{P_2 \mathbf{g} (\mathbf{g} \cdot \nabla \rho)} + \dots + \mathbf{f} \end{aligned}$$

Simplified Model

Toner, Tu, *Phys. Rev. Lett* (1995)

Continuity equation for density

$$\partial_t \rho = -\nabla \cdot \mathbf{g}$$

Simplification 1:

No fluctuations in
flocking direction

$$\delta \mathbf{g}_x = 0$$

Simplification 2:

No nonlinear
density terms

EOM for momentum density ($\mathbf{g} = g_0 \hat{x} + \mathbf{g}_\perp$, $\mathbf{g}_\perp = \delta \mathbf{g}_L + \delta \mathbf{g}_T$)

$$\begin{aligned} & \gamma \partial_t \mathbf{g}_\perp + \lambda_1 g_0 \partial_x \mathbf{g}_\perp + \lambda_1 \mathbf{g}_\perp \cdot \nabla_\perp \mathbf{g}_\perp + \lambda_2 \mathbf{g}_\perp \nabla_\perp \cdot \mathbf{g}_\perp + \lambda_3 \nabla_\perp (|\mathbf{g}_\perp|^2) \\ & = \underline{-\beta |\mathbf{g}_\perp|^2 \mathbf{g}_\perp} - \kappa_1 \nabla \rho + \mu_1 (\nabla_\perp^2 + \partial_x^2) \mathbf{g}_\perp + \mu_2 \nabla_\perp (\nabla_\perp \cdot \mathbf{g}_\perp) \\ & \quad + \mu_3 g_0^2 \partial_x^2 \mathbf{g} + \mathbf{f} \end{aligned}$$

3. Linear Theory

Linear Theory

- Expand EOM into $\mathbf{g} = g_0 \hat{\mathbf{x}} + \mathbf{g}_\perp$, $\mathbf{g}_\perp = \delta \mathbf{g}_L + \delta \mathbf{g}_T$, $\rho = \rho_0 + \delta \rho$
- Obtain propagators (response functions) $\mathbf{g}_\perp = \mathbf{G} \cdot \boldsymbol{\delta}$, $\delta \rho = H \delta$

$$\mathbf{G}_T = \frac{\mathcal{P}_T(\mathbf{q})}{-i\gamma\omega + i\lambda_1 g_0 q_x + \mu_x q_x^2 + \mu_\perp q_\perp^2}$$

$$\mathbf{G}_L = \frac{-i\omega \mathcal{P}_L(\mathbf{q})}{-i\omega(-i\gamma\omega + i\lambda_1 g_0 q_x + \mu_x q_x^2 + \mu_\perp^L q_\perp^2) + \kappa_1 q_\perp^2}$$

$$H = \frac{-i\gamma\omega + i\lambda_1 g_0 q_x + \mu_x q_x^2 + \mu_\perp^L q_\perp^2}{-i\omega(-i\gamma\omega + i\lambda_1 g_0 q_x + \mu_x q_x^2 + \mu_\perp^L q_\perp^2) + \kappa_1 q_\perp^2}$$

Linear Theory

- Scaling from equal-time correlation functions

$$\langle \delta \mathbf{g}_L(t, \mathbf{r}) \delta \mathbf{g}_L(t, \mathbf{r}') \rangle = D \int_{\mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{r}} \frac{\mathcal{P}_L(\mathbf{q})}{\mu_{\perp}^L q_{\perp}^2 + \mu_x q_x^2}$$

$$\langle \delta \mathbf{g}_T(t, \mathbf{r}) \delta \mathbf{g}_T(t, \mathbf{r}') \rangle = D \int_{\mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{r}} \frac{\mathcal{P}_T(\mathbf{q})}{\mu_{\perp} q_{\perp}^2 + \mu_x q_x^2}$$

$$\langle \delta \rho(t, \mathbf{r}) \delta \rho(t, \mathbf{r}') \rangle = \frac{D}{\kappa_1} \int_{\mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{r}} \frac{1}{\mu_{\perp}^L q_{\perp}^2 + \mu_x q_x^2}$$

- Typical linear scaling $\chi^{\text{lin}} = \chi_{\rho}^{\text{lin}} = \frac{2-d}{2}$

4. Nonlinear theory using FRG

Nonlinear theory using FRG

- Martin-Siggia-Rose-de Dominicis-Janssen → Ansatz for effective action
- Same couplings as in microscopic action
- Keep only relevant nonlinearities: $d_c = 4$

$$\begin{aligned}\Gamma[\bar{\mathbf{g}}_{\perp}, \mathbf{g}_{\perp}, \bar{\rho}, \rho] = & \int \{ \bar{\rho} [\partial_t \rho + \nabla_{\perp} \cdot \mathbf{g}_{\perp}] - D |\bar{\mathbf{g}}_{\perp}|^2 + \bar{\mathbf{g}}_{\perp} \cdot [\gamma \partial_t \mathbf{g}_{\perp} + \lambda_1 g_0 \partial_x \mathbf{g}_{\perp} \\ & + \lambda_1 \mathbf{g}_{\perp} \cdot \nabla_{\perp} \mathbf{g}_{\perp} + \lambda_2 \mathbf{g}_{\perp} \nabla_{\perp} \cdot \mathbf{g}_{\perp} + \lambda_3 \nabla_{\perp} (|\mathbf{g}_{\perp}|^2) + \beta |\mathbf{g}_{\perp}|^2 \mathbf{g}_{\perp} + \kappa_1 \nabla \rho \\ & - \mu_1 (\nabla_{\perp}^2 + \partial_x^2) \mathbf{g}_{\perp} - \mu_2 \nabla_{\perp} (\nabla_{\perp} \cdot \mathbf{g}_{\perp}) - \mu_3 g_0^2 \partial_x^2 \mathbf{g}_{\perp}] \}\end{aligned}$$

Regulator

- Sharp regulator for $\mathbf{q}_\perp$ only

$$R_k(\mathbf{q}_\perp, q_x, \omega) = \Gamma_k^{(2)}(\mathbf{q}_\perp, q_x, \omega) \left(\frac{1}{\Theta_\epsilon(|\mathbf{q}_\perp| - k)} - 1 \right)$$

- q_x and ω remain unregulated
→ ω and $\mathbf{q}_\perp$ Integrals can be performed analytically

Flow Equations

- Projection from Wetterich equation straightforward

$$\partial_k \Gamma_k = \frac{1}{2} \partial_{k'} \text{Tr} \log \left(\Gamma_k^{(2)} + R_{k'} \right) \Big|_{k'=k}$$

- Main challenge: Interaction between longitudinal and transverse modes

$$G_T = \frac{\mathcal{P}_T(\mathbf{q})}{-i\gamma\omega + i\lambda_1 g_0 q_x + \mu_\perp q_\perp^2 + \mu_x q_x^2}$$

$$G_L = \frac{-i\omega \mathcal{P}_L(\mathbf{q})}{-i\omega(-i\gamma\omega + i\lambda_1 g_0 q_x + \mu_\perp q_\perp^2 + \mu_x^L q_x^2) + \kappa_1 q_\perp^2}$$

→ Requires Computer algebra

Flow equations

Anomalous Dimensions $\rightarrow$ Anomalous Scaling

$$\partial_l D_k = \eta_{D,k} D_k$$

$$\partial_l \gamma_k = \eta_{\gamma,k} \gamma_k$$

$$\partial_l \mu_{\perp,k} = \eta_{\perp,k} \mu_{\perp,k}$$

$$\partial_l \mu_{x,k} = \eta_{x,k} \mu_{x,k}$$

Nonlinear Couplings

$$\partial_l \bar{\beta}_k = \left(4 - d - \frac{3}{2} \eta_{\perp,k} - \frac{1}{2} \eta_{x,k} \right) \bar{\beta}_k + \dots$$

$$\partial_l \bar{\lambda}_{i,k} = \frac{1}{4} (2(4 - d + \eta_{D,k} - \eta_{\gamma,k}) - 5\eta_{\perp,k} - \eta_{x,k}) \bar{\lambda}_{i,k} + \dots$$

2 Relevant Couplings

$$\partial_l \bar{\kappa}_{1,k} = (2 + \eta_{\gamma,k} - 2\eta_{\perp,k}) \bar{\kappa}_{1,k} + \dots$$

$$\partial_l \bar{\lambda}_{g,k} = \left(1 - \frac{1}{2} \eta_{\perp,k} - \frac{1}{2} \eta_{x,k} \right) \bar{\lambda}_{g,k} + \dots$$

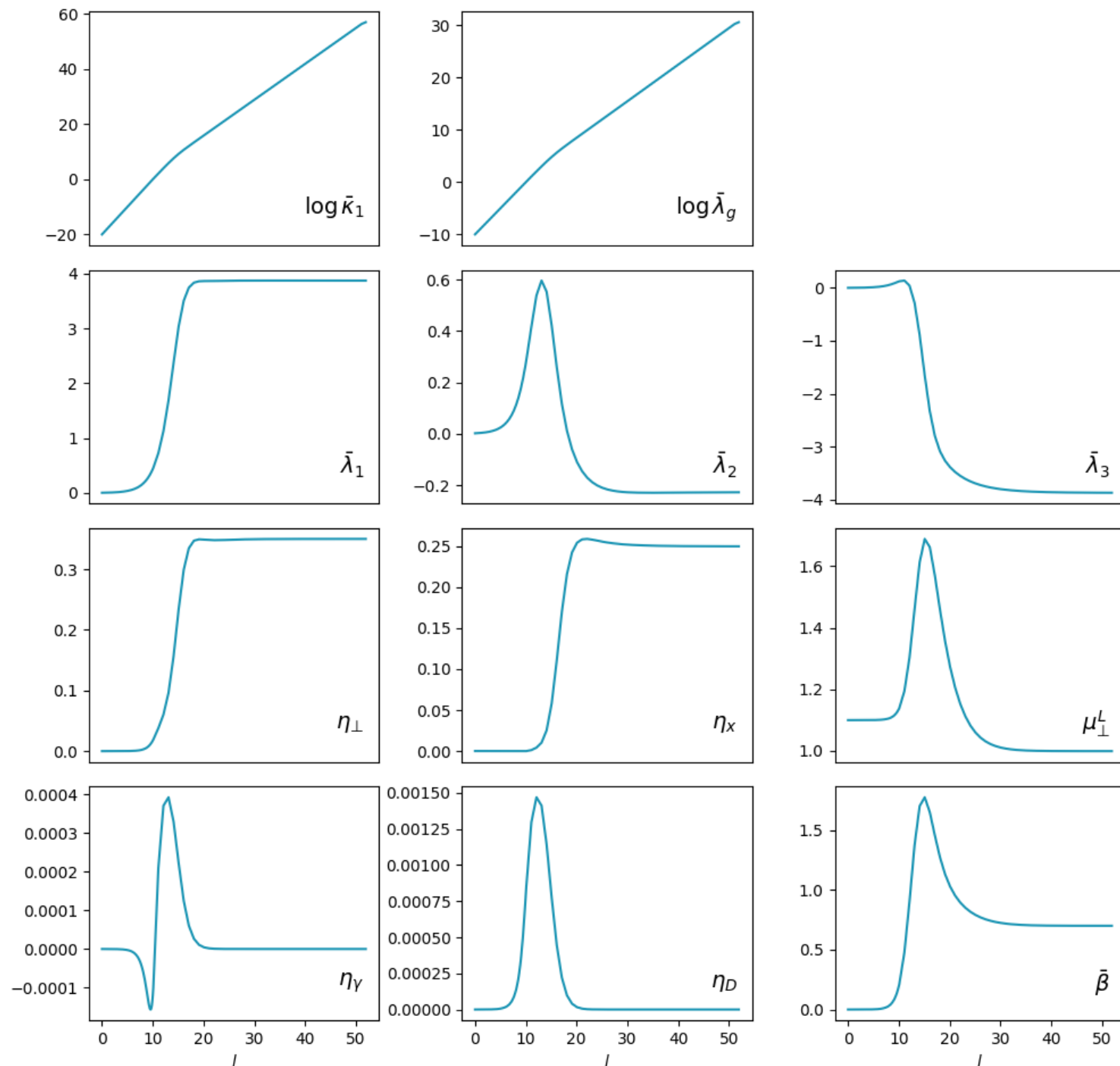
Marginal Linear Coupling

$$\partial_l \bar{\mu}_{\perp,k}^L = -\eta_{\perp,k} \bar{\mu}_{\perp,k}^L + \dots$$

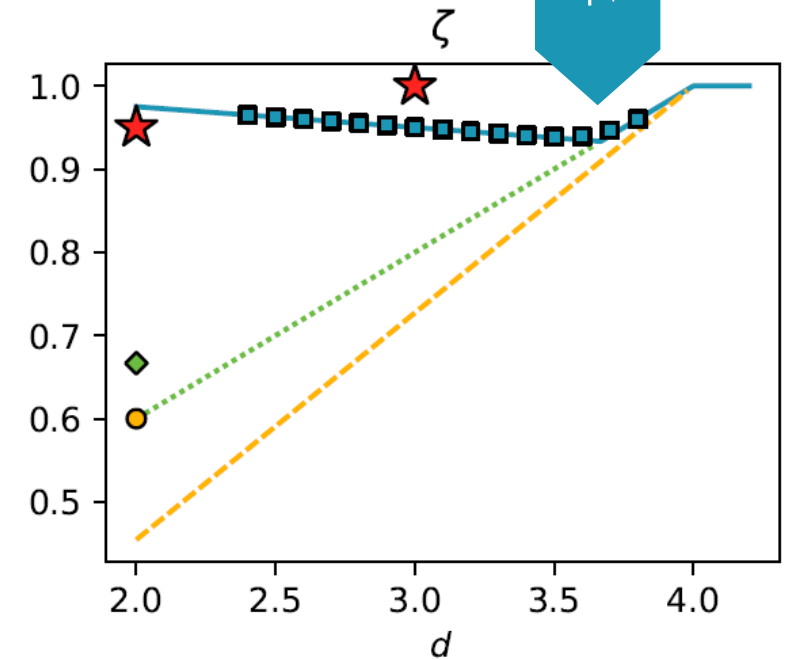
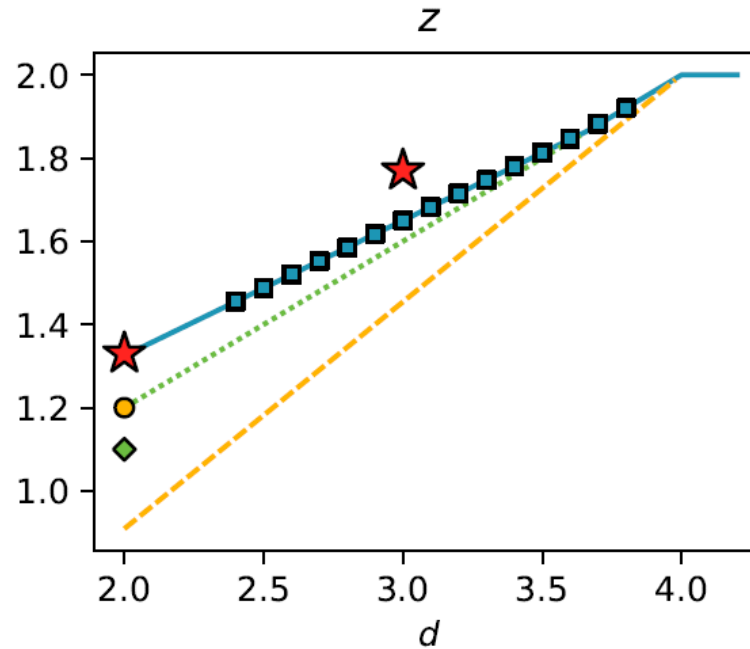
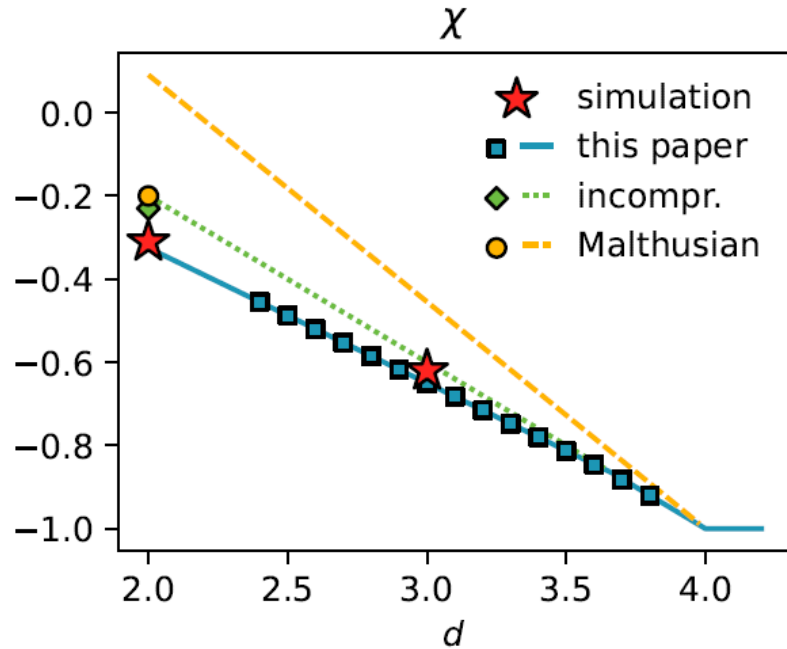
5. FRG fixed points

Numerical Integration of Flow Equations

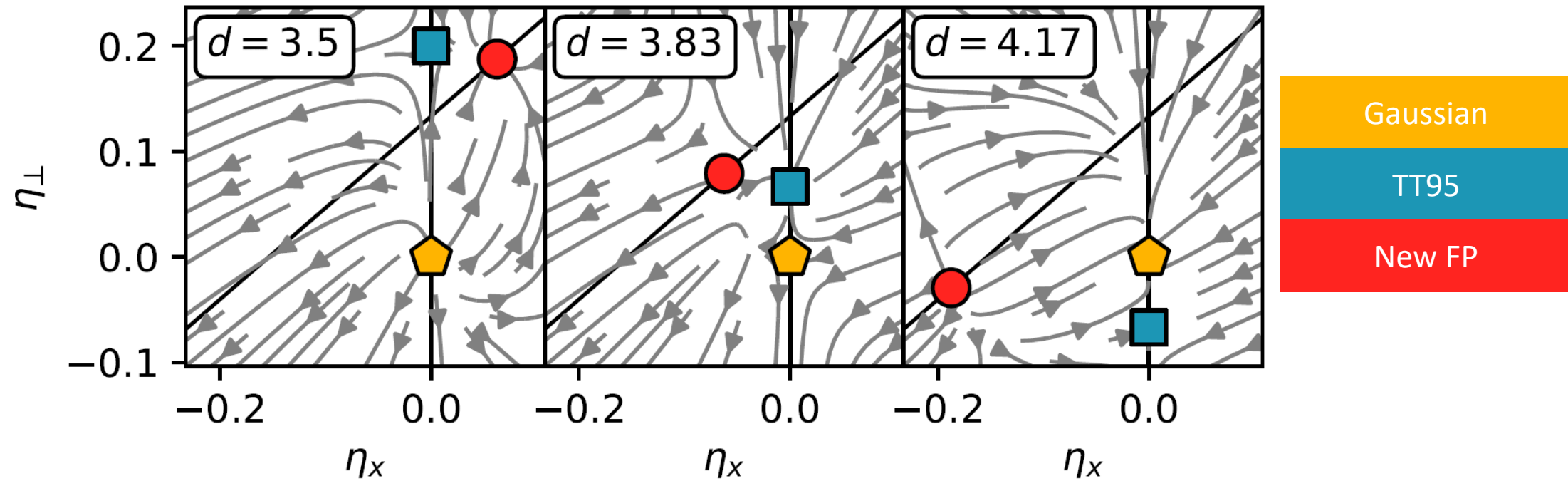
→ Fixed point



Scaling Exponents



Fixed points





6. Analytical treatment

Analytical Approach

- Full flow equations are intractable analytically
- Take limit of large relevant couplings
- Advantages are:
 1. Obtain analytical expressions for scaling exponents
 2. Why are exponents so accurate?
 3. Understand stability exchange

Limit of large relevant couplings

- $\bar{\kappa}_{1,k}$ and $\bar{\lambda}_{g,k}$ have positive scaling dimension $\rightarrow \infty$
- Assume $\bar{\kappa}_{1,k} \sim k^{-a}, \bar{\lambda}_{g,k} \sim k^{-b}$
- Evaluate Graphical corrections in this limit
- Set other couplings constant

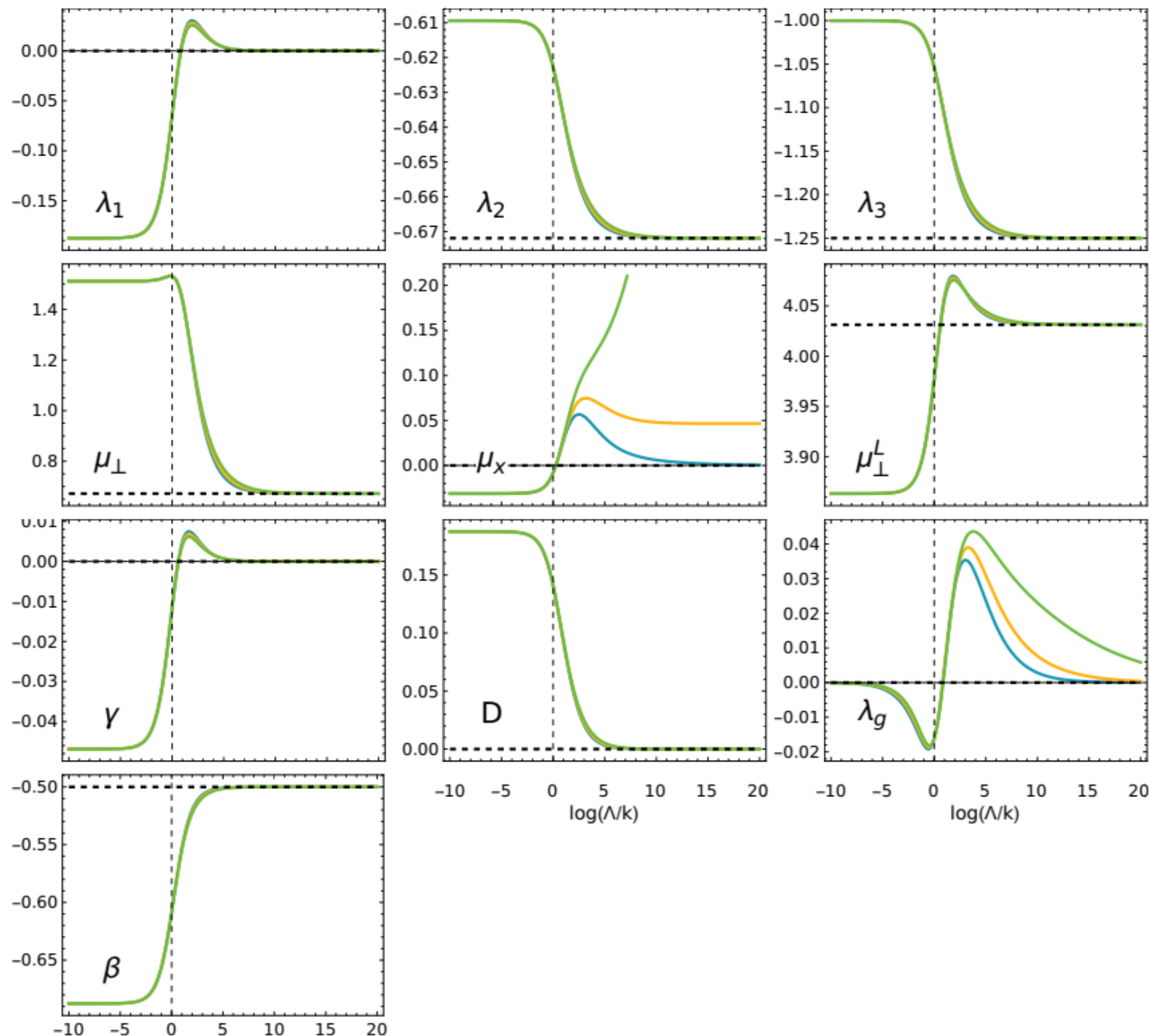
$$\partial_l \bar{\kappa}_{1,k} = (2 + \eta_{\gamma,k} - 2\eta_{\perp,k}) \bar{\kappa}_{1,k} + \dots$$

$$\partial_l \bar{\lambda}_{g,k} = \left(1 - \frac{1}{2}\eta_{\perp,k} - \frac{1}{2}\eta_{x,k}\right) \bar{\lambda}_{g,k} + \dots$$

Graphical corrections

- For large mass-scales
- Evaluated analytically and numerically

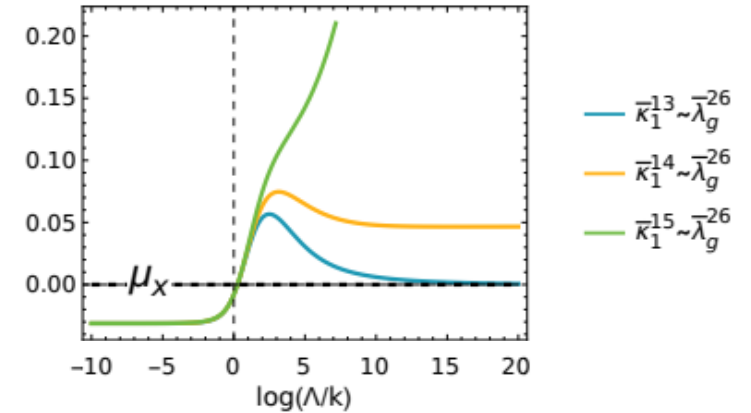
— $\bar{\kappa}_1^{13} \sim \bar{\lambda}_g^{26}$
— $\bar{\kappa}_1^{14} \sim \bar{\lambda}_g^{26}$
— $\bar{\kappa}_1^{15} \sim \bar{\lambda}_g^{26}$



Unprecedented scaling relation

$$\eta_{x,k} = \frac{1}{\mu_{x,k}} \partial_l \mu_{x,k} = F(\bar{\lambda}_{1,k}, \bar{\lambda}_{2,k}, \bar{\lambda}_{3,k}, \bar{\lambda}_{g,k}, \bar{\kappa}_{1,k})$$

$$\rightarrow F\left(\bar{\lambda}_{1,k}^*, \bar{\lambda}_{2,k}^*, \bar{\lambda}_{3,k}^*, \frac{\bar{\lambda}_{g,k}^{13}}{\bar{\kappa}_{1,k}^7}\right)$$



$$F(\bar{\lambda}_{1,k}^*, \bar{\lambda}_{2,k}^*, \bar{\lambda}_{3,k}^*, 0) = 0 \quad \rightarrow \eta_x = 0 \quad \rightarrow \text{TT Univsersality Class}$$

$$F(\bar{\lambda}_{1,k}^*, \bar{\lambda}_{2,k}^*, \bar{\lambda}_{3,k}^*, \infty) = \infty \quad \rightarrow \text{No FP}$$

$$F(\bar{\lambda}_{1,k}^*, \bar{\lambda}_{2,k}^*, \bar{\lambda}_{3,k}^*, \text{const}) = \text{const} \quad \rightarrow \bar{\lambda}_{\kappa,k} = \frac{\bar{\lambda}_{g,k}^{13}}{\bar{\kappa}_{1,k}^7} = \text{const}$$

$$\rightarrow \partial_l \bar{\lambda}_{\kappa,k} = [13(z - \zeta) - 7(2z - 2)] \bar{\lambda}_{\kappa,k} = 0$$

Analytical expression for exponents

$$\partial_l \bar{\lambda}_{\kappa,k} = 0 \rightarrow 13(z - \zeta) - 7(2z - 2) = 0$$

$$\partial_l \lambda_{1,k} = 0 \rightarrow z - 1 + \chi = 0$$

$$\partial_l D_k = 0 \rightarrow z - 2\chi - \zeta - (d - 1) = 0$$

$$\rightarrow \chi = \frac{13(1 - d)}{40}, \quad z = \frac{27 + 13d}{40}, \quad \zeta = \frac{41 - d}{40}$$

3 exponents are constrained by 3 scaling relations:
Conjecture: Robust exponents with no truncation dependence

Stability of Fixed point

Expanding around TT Fixed point:

$$\partial_l \bar{\lambda}_{\kappa,k} = [13(z - \zeta) - 7(2z - 2)] \bar{\lambda}_{\kappa,k}$$

$$\chi = \frac{3 - 2d}{5}, \quad z = \frac{2(d + 1)}{5}, \quad \zeta = \frac{d + 1}{5}$$

TT Fixed point becomes unstable below $d = \frac{11}{3}$

TT and our exponents agree in $d = \frac{11}{3}$

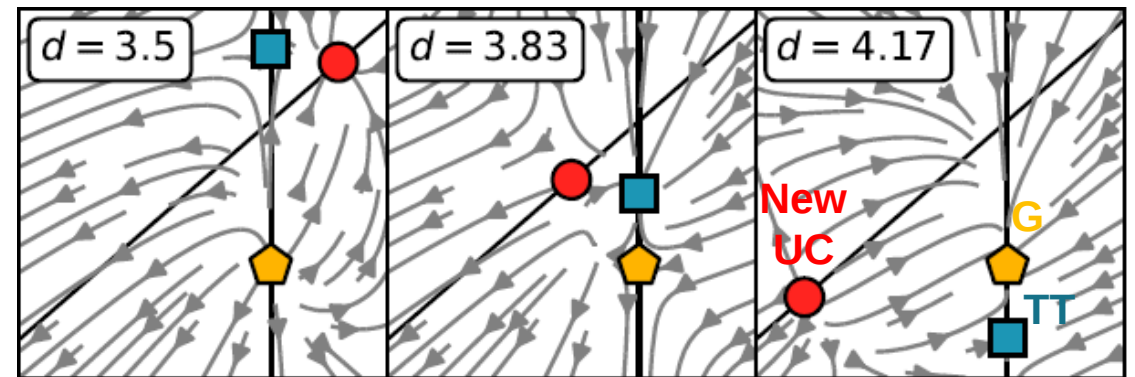
Supports conjecture of stability exchange



7. Summary and Outlook

Summary

- We used nonperturbative, functional RG to study a simplified TT model
- TT UC applies for $\frac{11}{3} (\approx 3.67) < d < 4$
- Below $d = 11/3$, a new UC emerges, whose scaling exponents agree remarkably well with simulation in 2D & 3D



Spatial dimension (d)	χ	z	ζ
$d = 2 :$			
TT 95	-0.20	1.20	0.6
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new UC	-0.65	1.65	0.95

Outlook: Active Matter - a treasure trove new UCs

Marchetti, Joanny, Ramaswamy, Liverpool, Prost, Rao, & Aditi Simha

Hydrodynamics of soft active matter

Rev. Mod. Phys. 85, 1143 (2013)

“The hope is to be able to classify active matter in a small number of universality classes, based on considerations of symmetry and conservation laws, each with a well-defined macroscopic behavior. We consider here four classes of active matter according to the nature of the broken symmetry of the ordered phase and the type of momentum damping.”

TABLE I. Examples of active systems classified according to symmetry and to the relevance of momentum conservation (wet or dry).

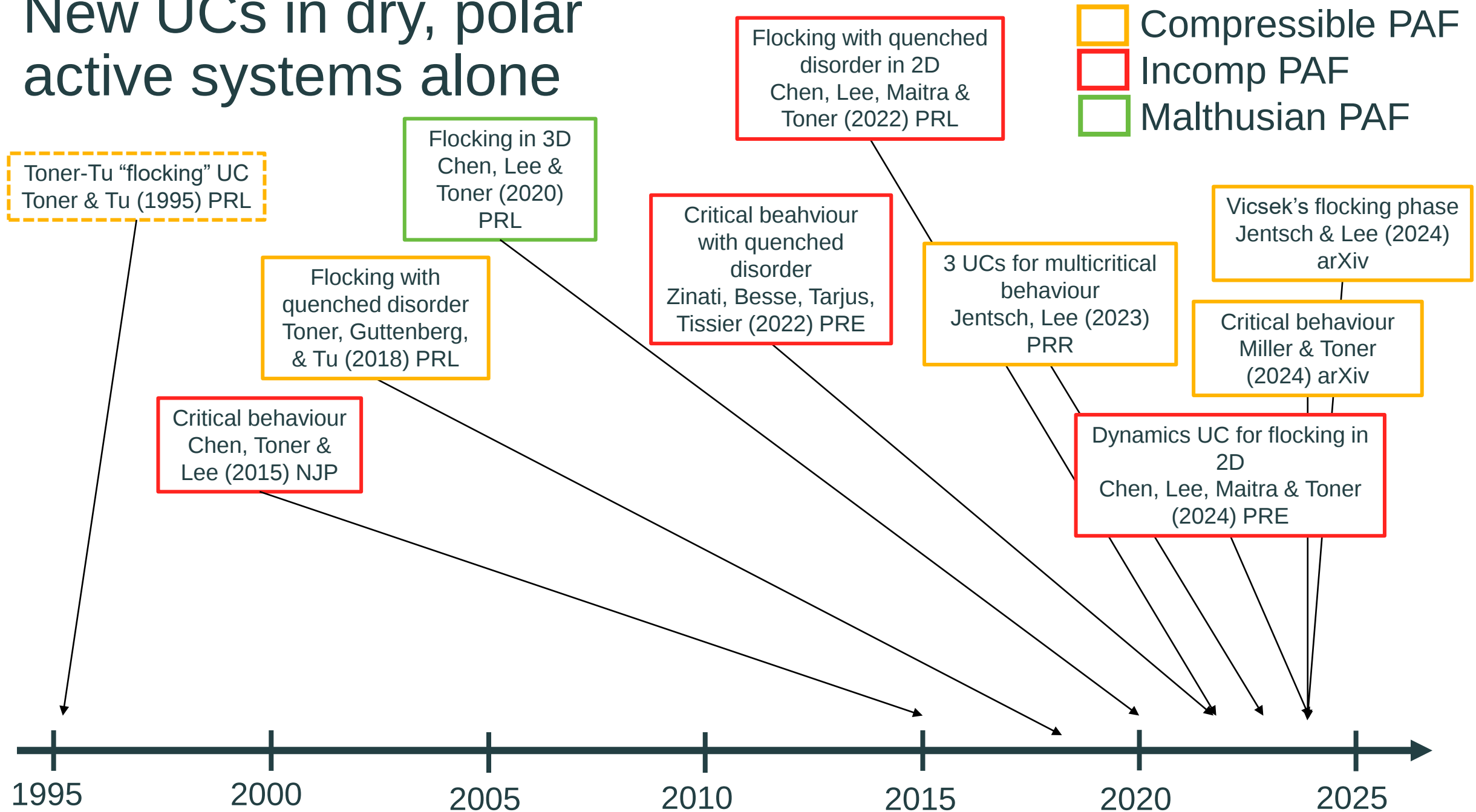
	Nematic	Polar
Dry	Melanocytes (Kemkemer <i>et al.</i> , 2000) Vibrated granular rods (Narayan, Ramaswamy, and Menon, 2007)	Migrating animal herds (Parrish and Hamner, 1997) Migrating cell layers (Serra-Picamal <i>et al.</i> , 2012) Vibrated asymmetric granular particles (Kudrolli <i>et al.</i> , 2008) Films of cytoskeletal extracts (Surrey <i>et al.</i> , 2001)
Wet	Suspensions of catalytic colloidal rods (Paxton <i>et al.</i> , 2004)	Cell cytoskeleton and cytoskeletal extracts in bulk suspensions (Bendix <i>et al.</i> , 2008) Swimming bacteria in bulk (Dombrowski <i>et al.</i> , 2004) Pt catalytic colloids (Palacci <i>et al.</i> , 2010)

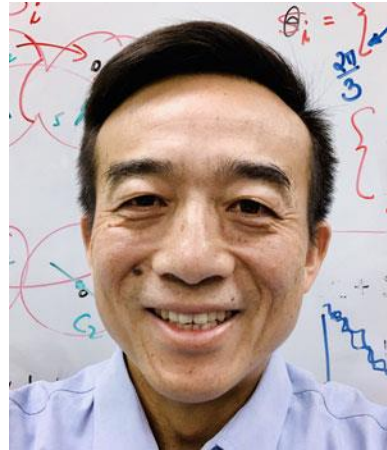
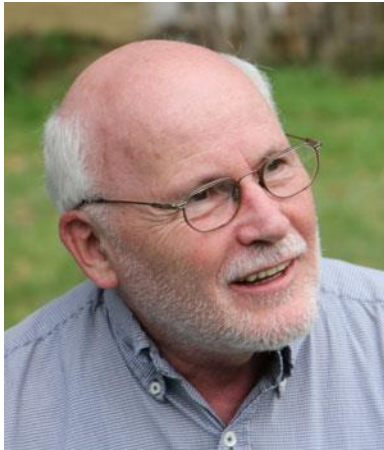
New UCs in dry, polar active systems alone

Toner-Tu “flocking” UC
Toner & Tu (1995) PRL



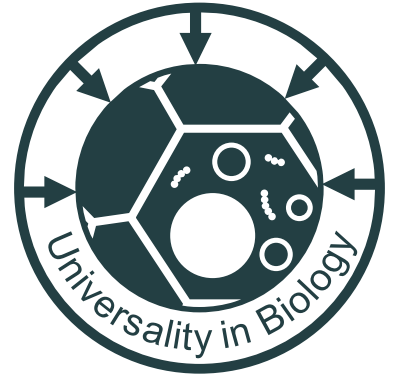
New UCs in dry, polar active systems alone





No one will drive us from the paradise which Toner, Vicsek, & Tu created for us.

Outlook 2



5 decades ago, technological relevance, experimental advances, and abundance of novel physics propelled condensed matter physics to become the 'King of Physics' [Martin (2019) Physics Today]

With its relevance to health and life, experimental advances, and abundance of novel physics, I believe living matter physics will be the next 'condensed matter physics'