SoftBio Theory Seminar, Physics Department
Oxford University, 13 November 2023

Active Matter

A treasure trove of novel universality classes

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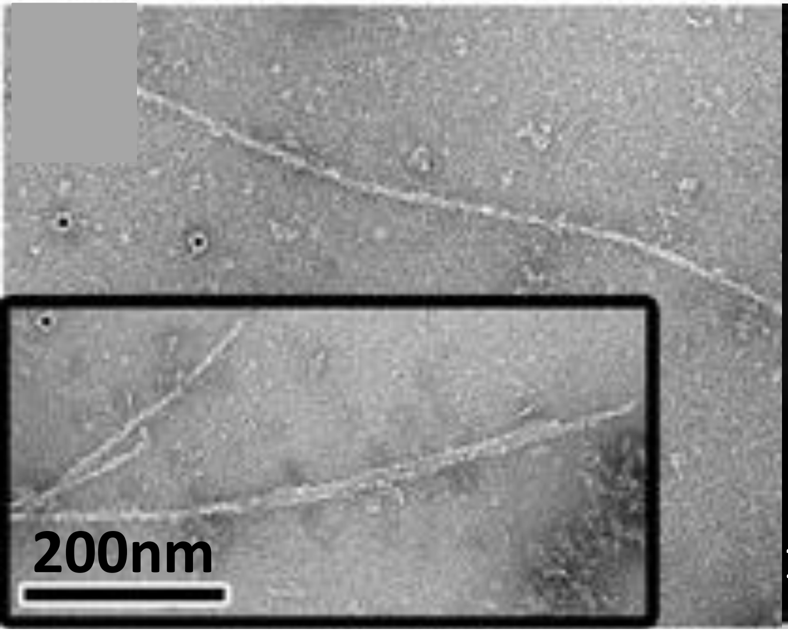
Universality in Biology

Universal physics in biology

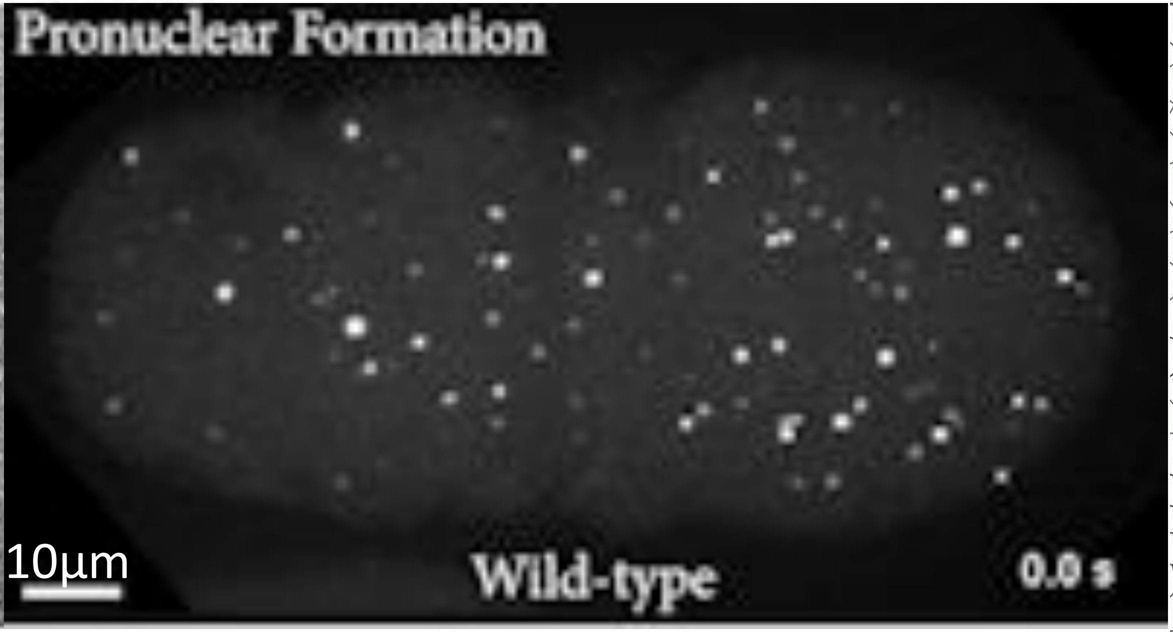
Universality in Biology

Amyloid formation



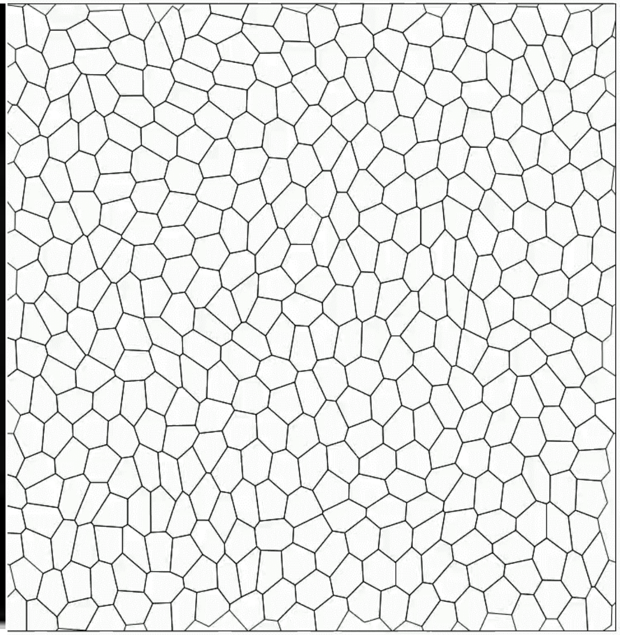
Pytowski, Lee, Foley, Vaux, Jean (2020)
Liquid–liquid phase separation of type II diabetes associated IAPP initiates hydrogelation and aggregation
PNAS

Biomolecular condensation



Folkmann, Putnam, Lee, Seydoux (2021)
Regulation of biomolecular condensates by interfacial protein clusters
Science

Active matter



Killeen, Bertrand, Lee (2022)
Polar Fluctuations Lead to Extensile Nematic Behavior in Confluent Tissues
Phys Rev Lett

Acknowledgement

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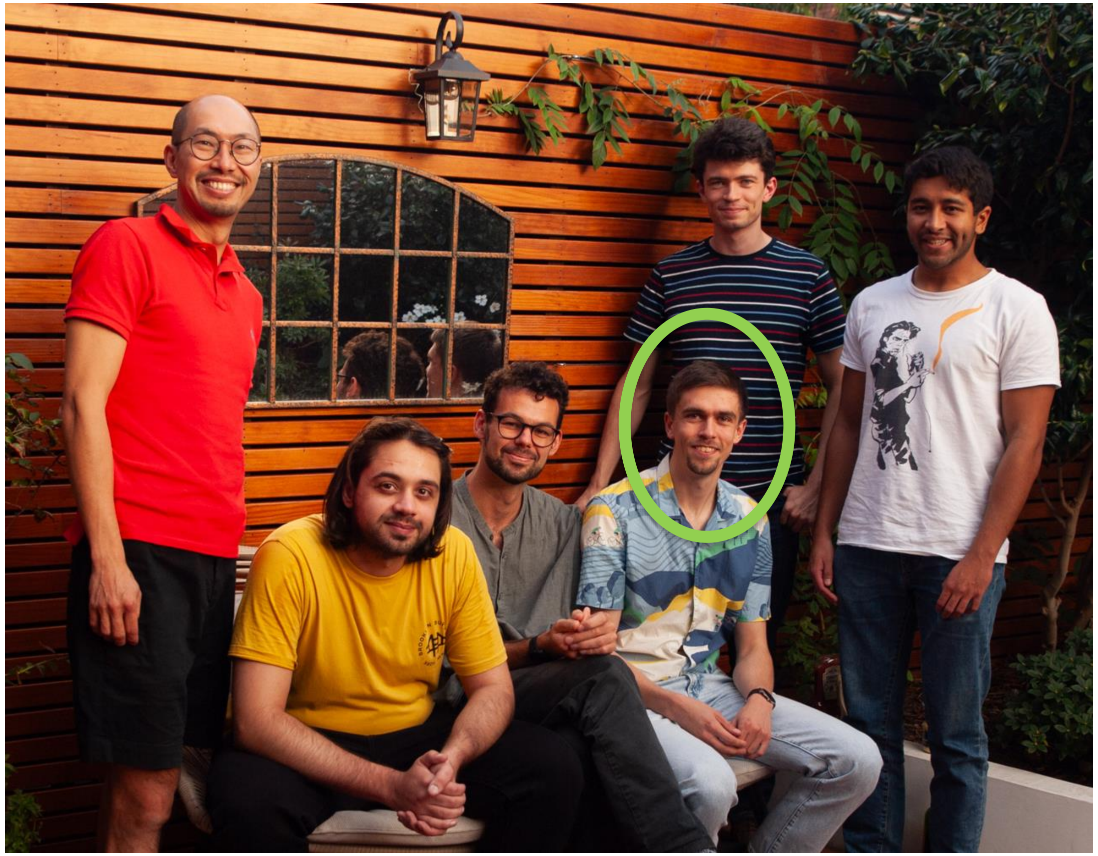
Patrick Jentsch



Leiming Chen
(China U Mining & Technology)

Ananyo Maitra
(CY Cergy Paris Université)

John Toner
(Oregon)

Imperial College London

EPSRC
Engineering and Physical Sciences Research Council

BBSRC
bioscience for the future

The Leverhulme Trust

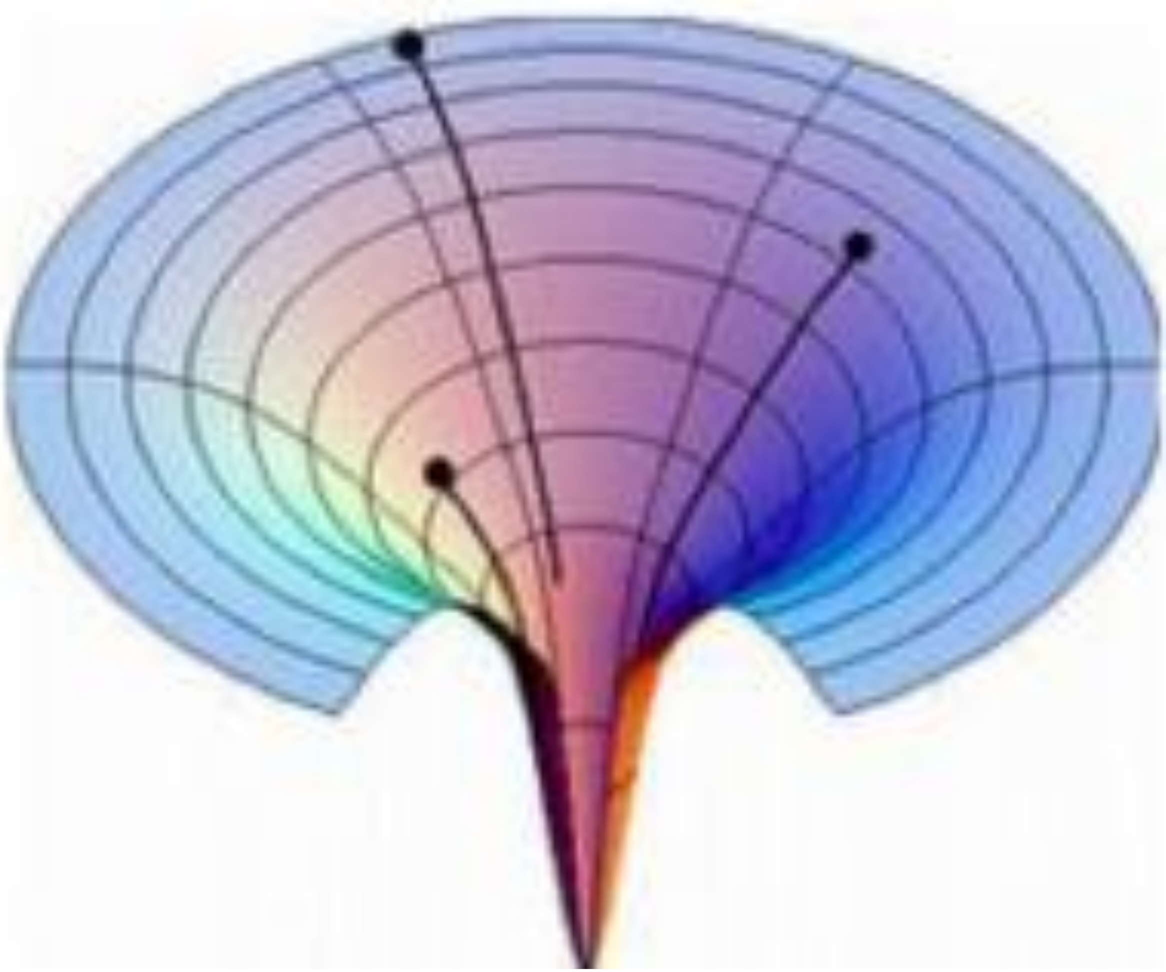
CANCER RESEARCH UK | IMPERIAL CENTRE

NIHR | Imperial Biomedical Research Centre

Plan of talk

1. What are universality classes (UC)?
2. Active matter: Incompressible polar active fluids (IPAF)
3. New UC in IPAF
4. Beyond IPAF
5. Summary & Outlook

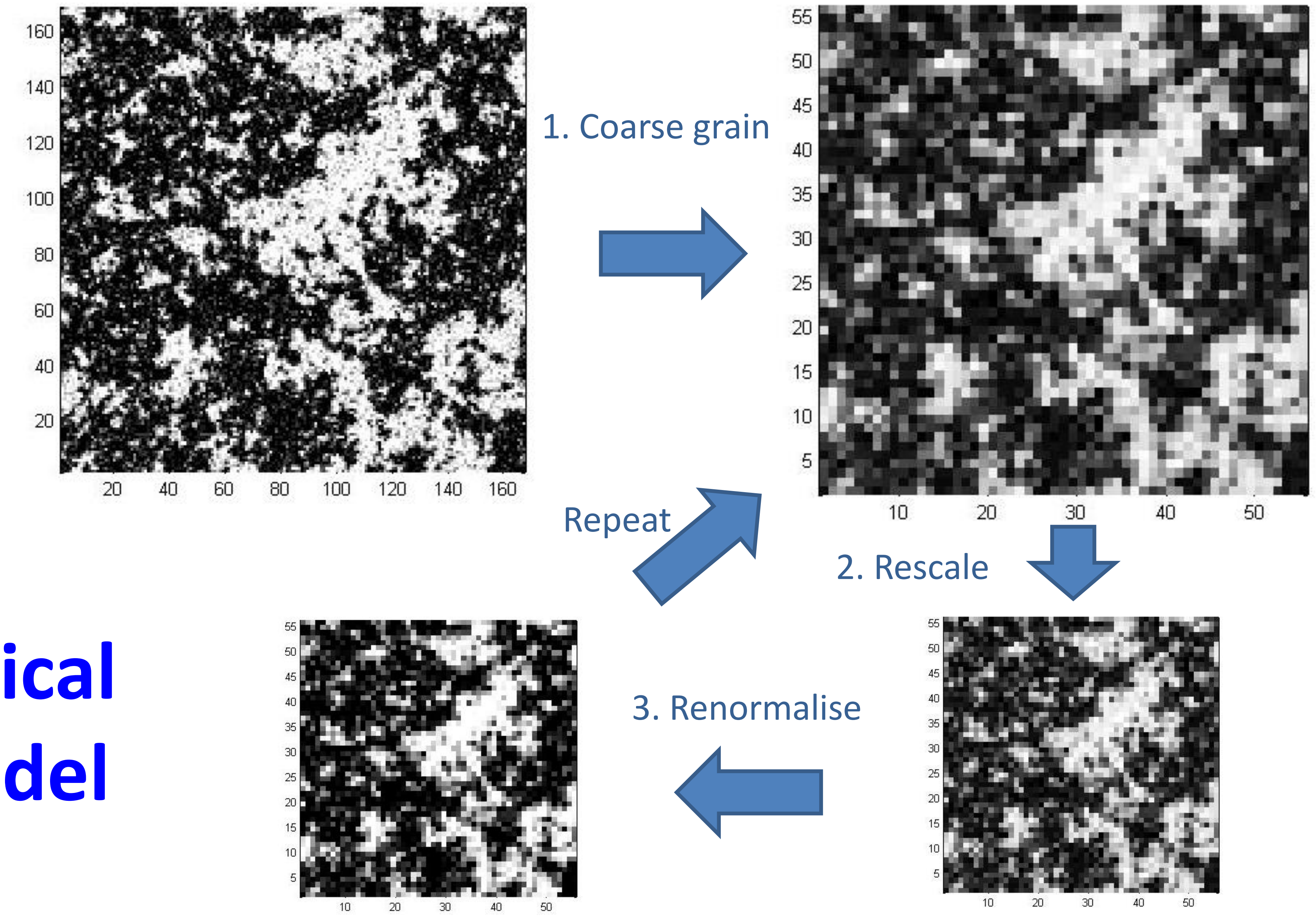
1. What are universality classes (UC)?



UC = renormalisation group fixed point

- What is the Renormalization Group (RG)?

- RG is
  - a coarse-graining procedure
  - incorporates short-distance dynamics into longer-distance dynamics



E.g., Critical Ising model

Critical Ising: RG flow = Parameter flow of Hamiltonian

$$H_0 = \int d^d r \left[(\nabla S(\mathbf{r}))^2 + t_0 S(\mathbf{r})^2 + u_0 S(\mathbf{r})^4 + g_6 S(\mathbf{r})^6 + \lambda_4 (\nabla S(\mathbf{r}))^4 + \ldots \right]$$

1. Coarse grain out short wavelength fluctuations
2. Rescale
3. Renormalise

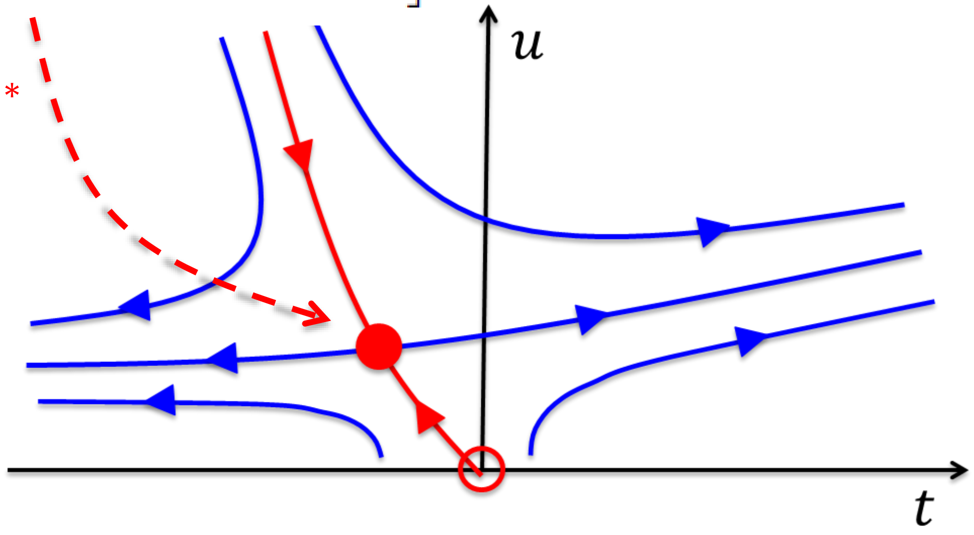
$$H_l = \int d^d r \left[(\nabla S(\mathbf{r}))^2 + t_l S(\mathbf{r})^2 + u_l S(\mathbf{r})^4 \right]$$

l goes up further

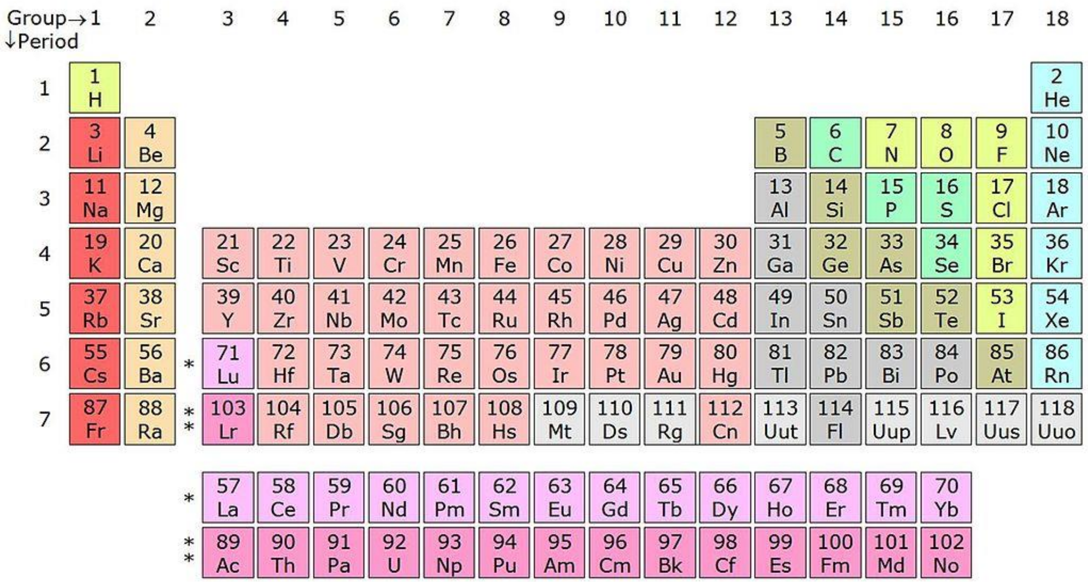
$$H^* = \int d^d r \left[(\nabla S(\mathbf{r}))^2 + t^* S(\mathbf{r})^2 + u^* S(\mathbf{r})^4 \right]$$

l : Level of coarse-graining

Parameter-free model H^* becomes the (asymptotically) exact model



Categorisation: chemical elements vs. universality classes

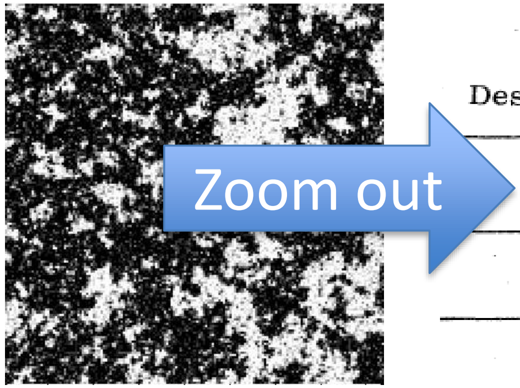


Group→ ↓Period	1	2		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca		21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr		39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo

*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb
**	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No

Atomic no. → elements

Hohenberg and Halperin (1977) Rev. Mod. Phys.



Designation	System
A	Kinetic Ising anisotropic magnets
B	Kinetic Ising uniaxial ferromagnet
C	Anisotropic magnets structural transition
H	Gas–liquid binary fluid
E	Easy-plane magnet, $h_z = 0$
F	Easy-plane magnet, $h_z \neq 0$ superfluid helium
G	Heisenberg antiferromagnet
J	Heisenberg ferromagnet

Symmetries & conservation laws → UC

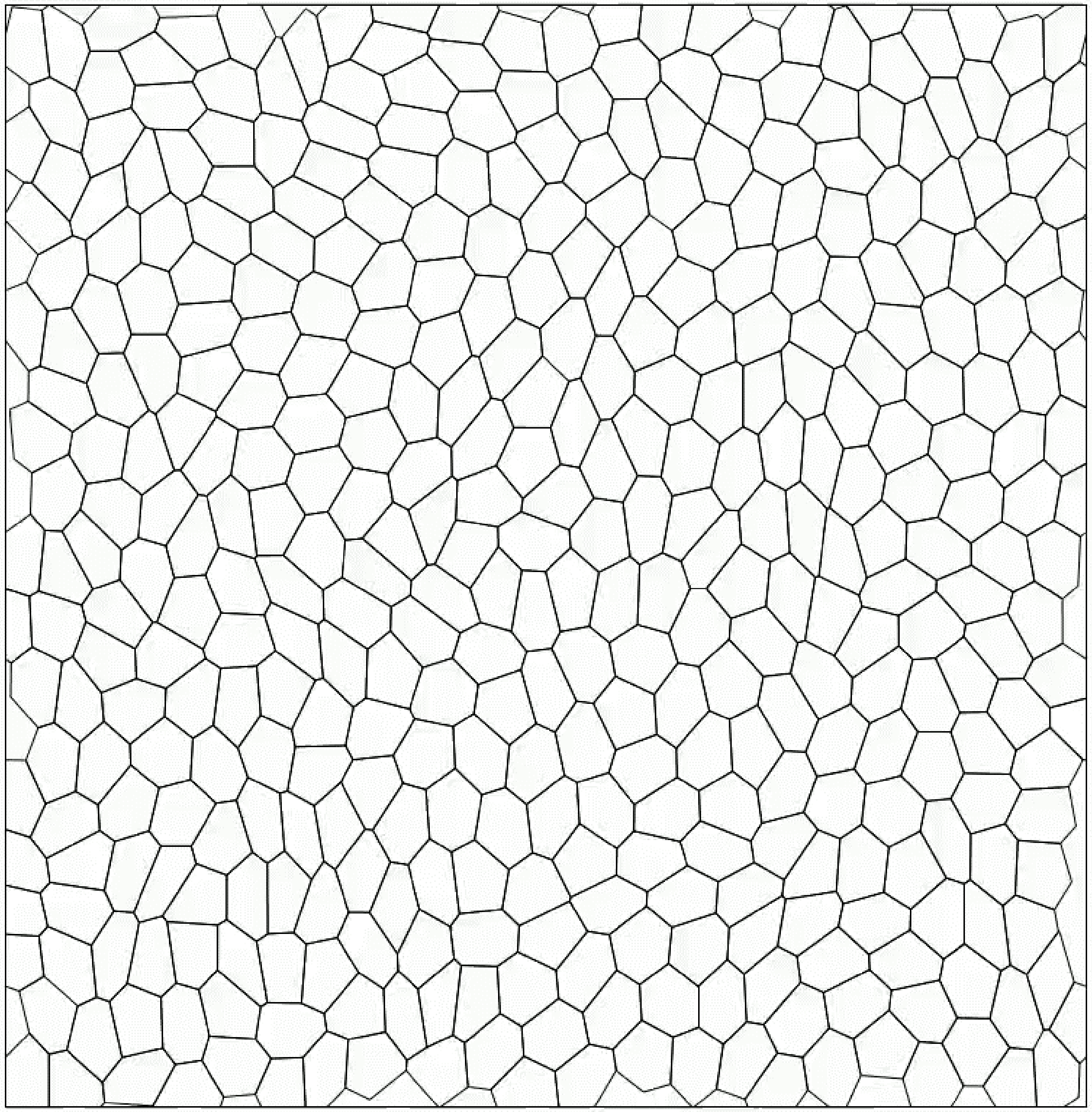
Universality classes (UCs)

- Are fixed points of RG transformation

- Categorise dynamical systems into distinct critical phenomena & phases (states of matter)

- Provide unambiguous definition of novel physics
  - New UC → New physics

2. Active matter: Incompressible polar active fluids (IPAF)



Q: Given how complicated biological systems are (e.g., cell tissues), how is meaningful modelling ever possible?

A: Even if the microscopic dynamics is complicated, the underlying symmetry & conservation law may be simple

→ *universal hydrodynamic equations of motion*

E.g., Navier-Stokes for thermal fluids

- Incompressible limit for simplicity
- Hydrodynamic variable: velocity $\vec{v}$
- Equation of motion (EOM):

$$\partial_t \vec{v} = \rho^{-1} \vec{F} \text{ , with constraint } \nabla \cdot \vec{v} = 0$$

- What is the force field $\vec{F}$?

Symmetries

$\boxed{\text{EOM: } \partial_t \vec{v} = \rho^{-1} \vec{F}}$

- Temporal invariance: **F** does not depend on time
- Translational invariance: **F** does not depend on position **r**
- Rotational invariance: **F** does not depend on a particular direction
- Chiral (parity) invariance: **F** is not right-handed or left-handed
- Galilean invariance: EOM invariant under a boost

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- To order $O(v^2, \partial^2)$ [hydrodynamics], incompressible NS EOM:

$$\partial_t \vec{v} = -\vec{\nabla} P - (\vec{v} \cdot \vec{\nabla})\vec{v} + \mu \nabla^2 \vec{v} + \cdots + \vec{f_0}$$

‘pressure’ (Lagrange multiplier) to enforce incompressibility

Noise term satisfying symmetries & fluctuation-dissipation relation

IPAF

$\boxed{\text{EOM: } \partial_t \vec{v} = \rho^{-1}\vec{F}}$

- Temporal invariance: **F** does not depend on time
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- ~~Galilean invariance: EOM invariant under a boost~~
- Noise term satisfying symmetries ~~& fluctuation-dissipation relation~~

Hydrodynamic EOM of incompressible active fluids

$$\partial_t \vec{v} = -\vec{\nabla} P + \overrightarrow{f_0} - \lambda_0 (\vec{v} \cdot \vec{\nabla}) \vec{v} - (a_0 + b_0 v^2) \vec{v} - \mu_0 \nabla^2 \vec{v} + c_0 v^4 \vec{v} + \xi_0 (\nabla^2)^2 \vec{v} + \cdots$$

Gaussian noise

Higher order terms in ∇ & $\vec{v}$

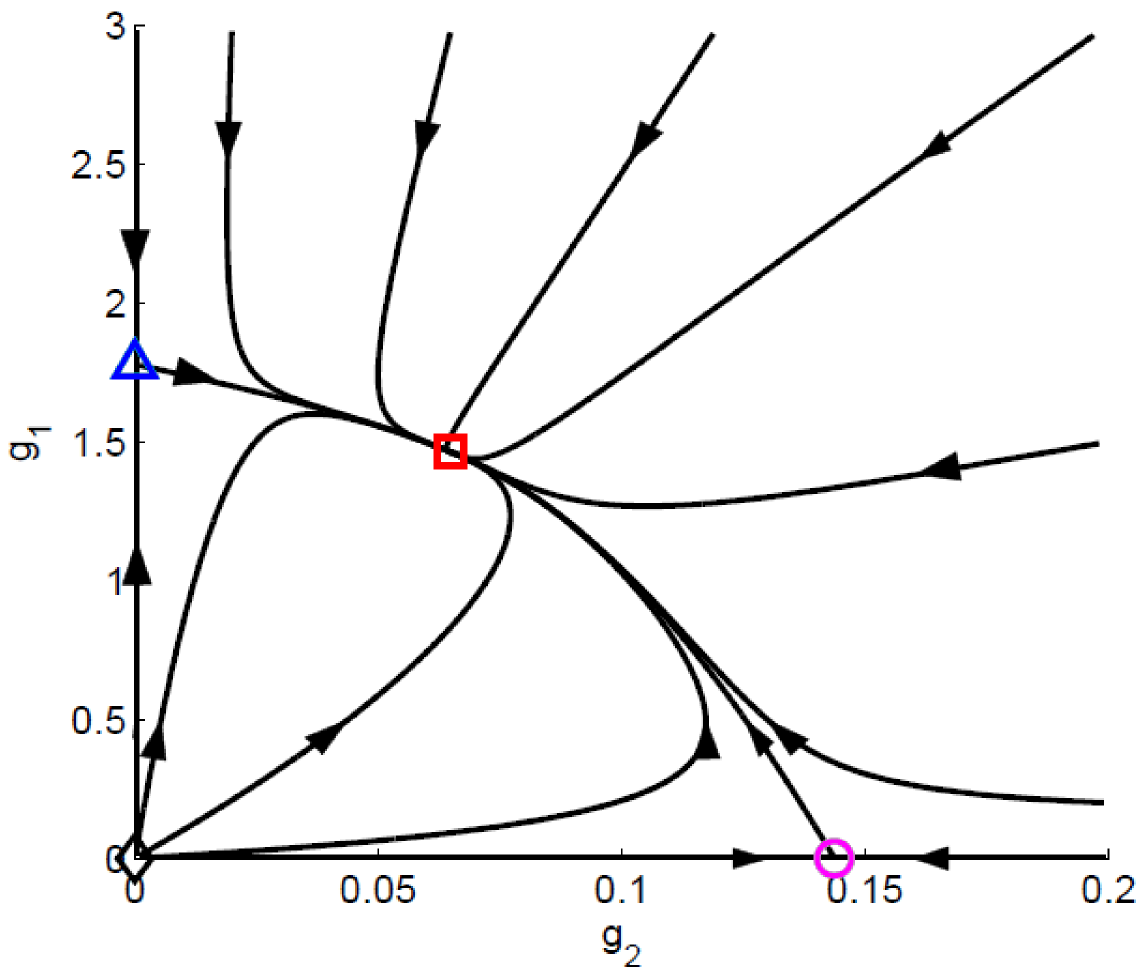
‘pressure’ (Lagrange multiplier) to enforce incompressibility

Now that we have a model (a.k.a. Toner-Tu model [Toner & Tu (1995) PRL]), what do we do next?

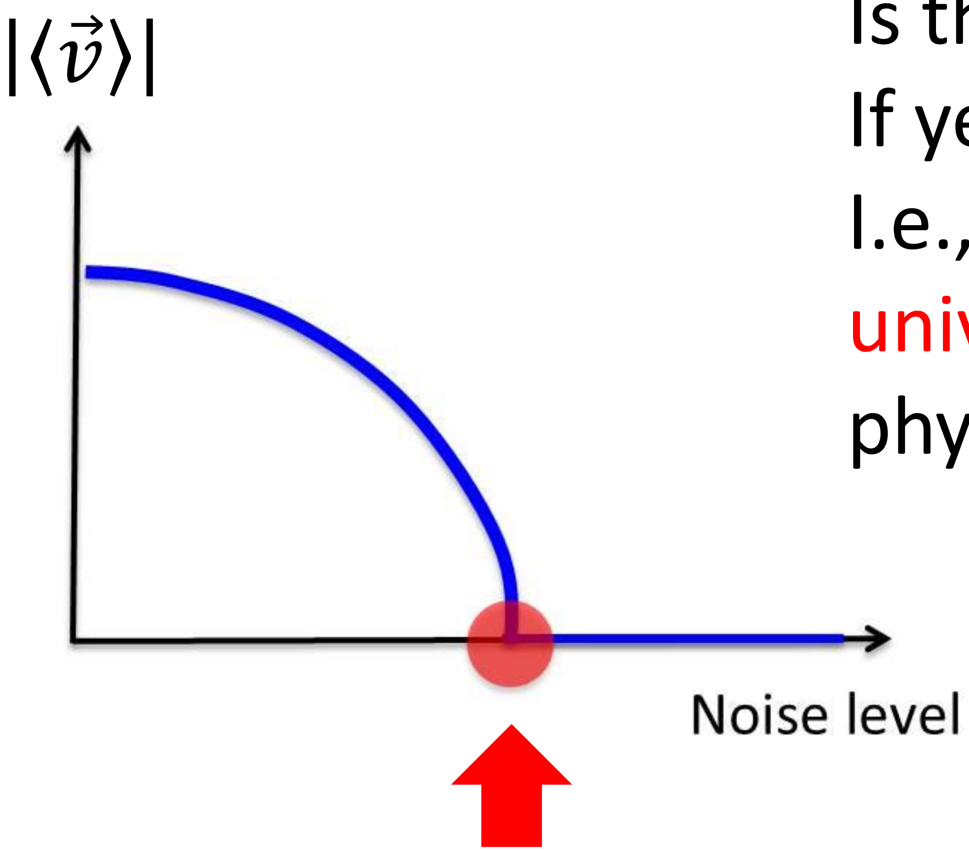
Simplest thing first → mean-field analysis: $(a_0 + b_0 v^2) = 0$

$a_0 < 0$ → ordered phase with non-zero $\vec{v}$: **active** collective motion

3. New UCs in IPAF



Order-disorder critical transition



Critical region

Is the transition really critical?
If yes, is it a new critical phenomenon?
I.e., does it correspond to a new universality class in non-equilibrium physics?

RG analysis of the EOM

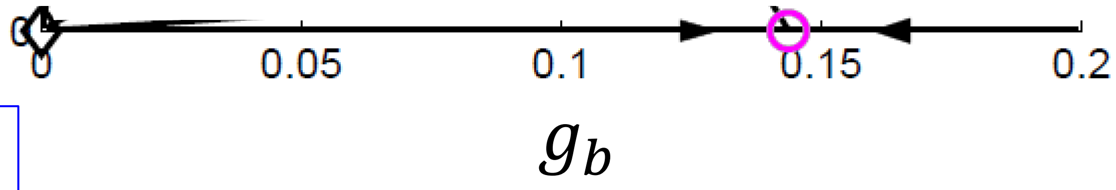
$$\partial_t \vec{v} + \vec{\nabla} P + \vec{f_l} = -\lambda_l (\vec{v} \cdot \vec{\nabla}) \vec{v} - (a_l + b_l v^2) \vec{v} - \mu_l \nabla^2 \vec{v}$$

Two nonlinearities: λ_l and b_l, $l \sim$ level of RG transformation

Methodology:

Dynamic RG + ϵ-expansion at 1-loop

[L Chen, J Toner, CFL (2015) New J. Phys. 17, 042002]

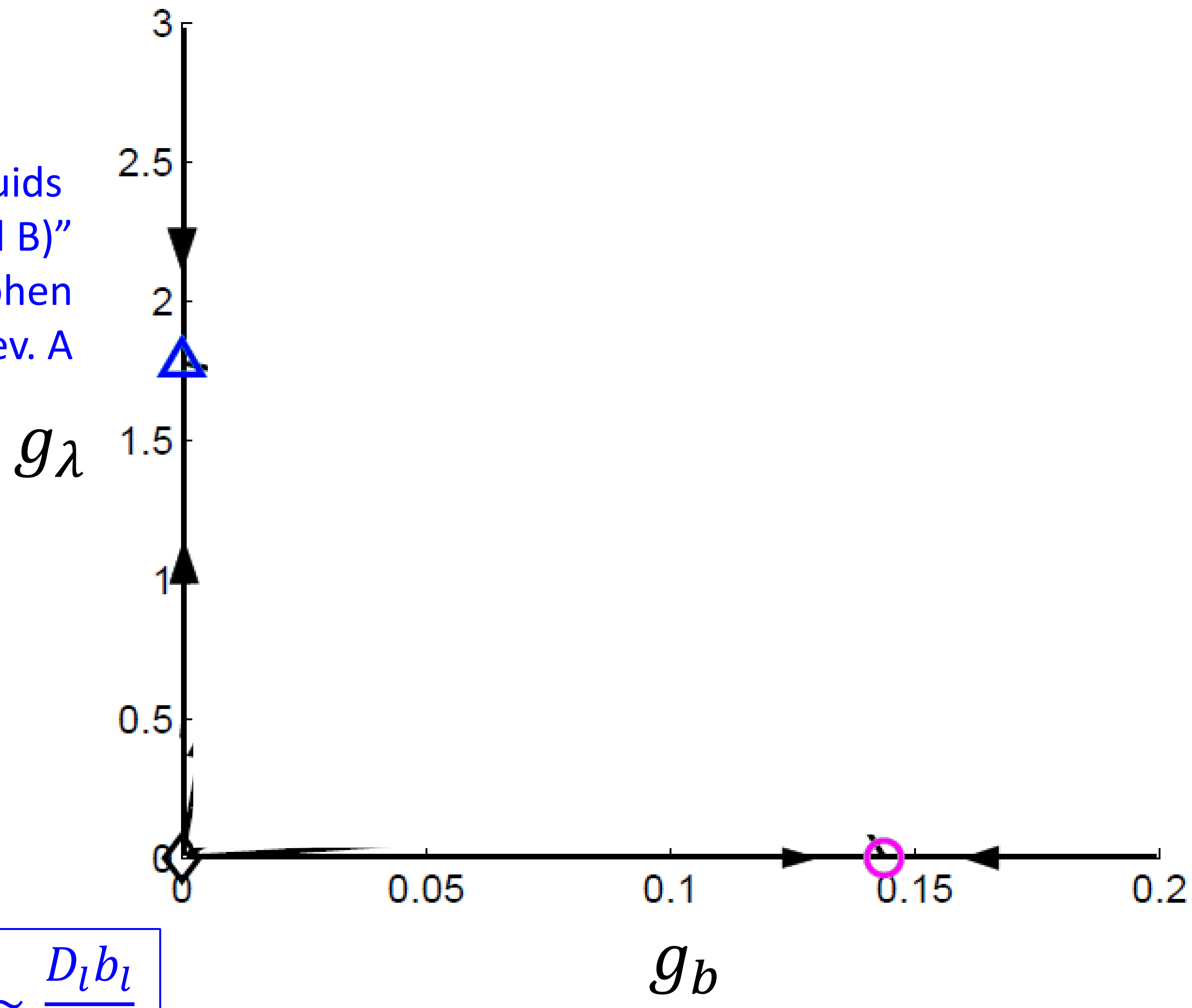


“Ferromagnets with dipolar interactions”
Aharony and Fisher (1973) Phys. Rev. Lett.

$$\boxed{g_\lambda(l) \sim \frac{D_l {\lambda_l}^2}{{\mu_l}^3};\ g_b\,(l) \sim \frac{D_l b_l}{{\mu_l}^2}}$$

L Chen, J Toner, CFL (2015)
New J. Phys. 17, 042002

“Randomly stirred fluids (Model B)”
Forster, Nelson & Stephen (1977) Phys. Rev. A



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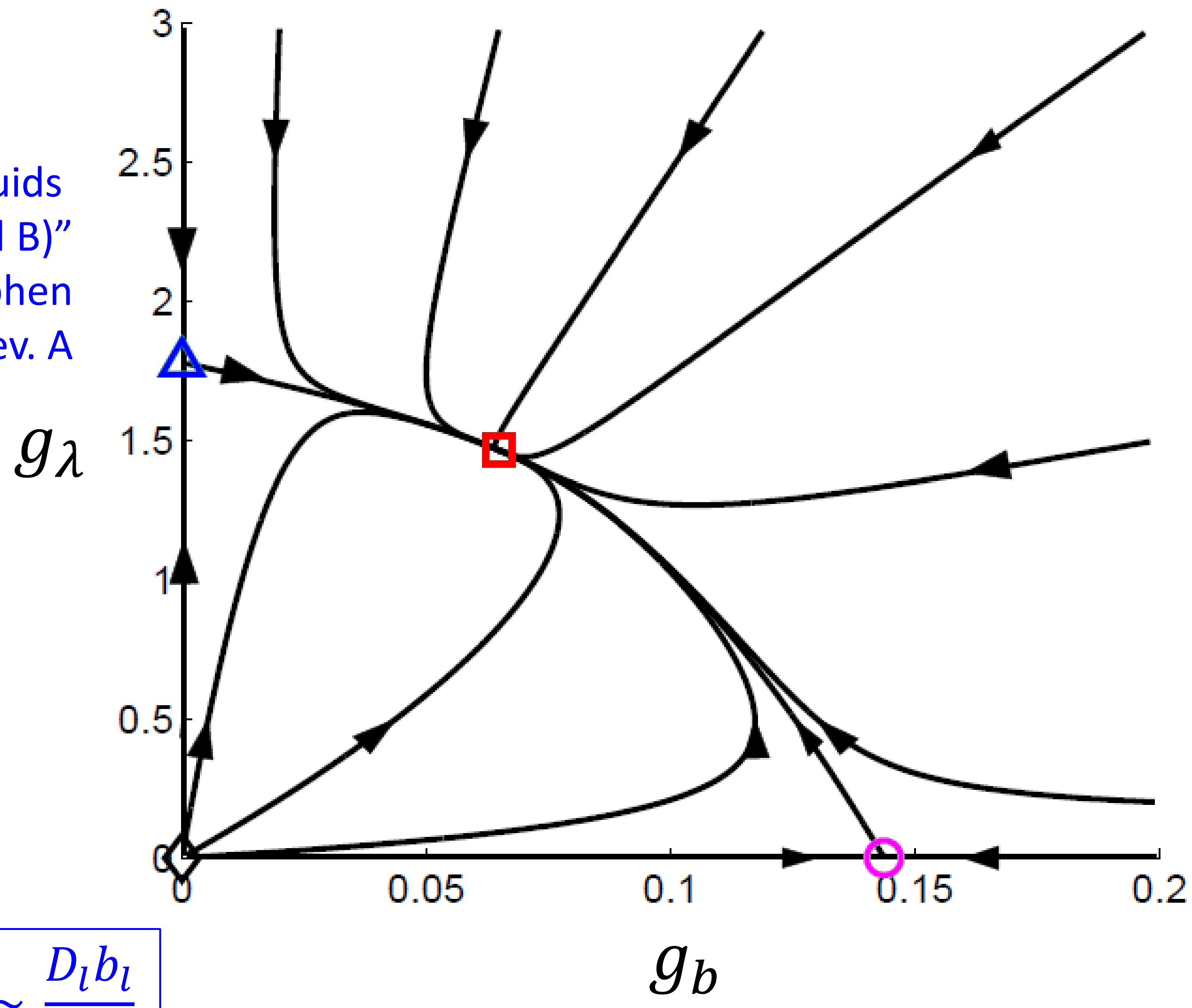

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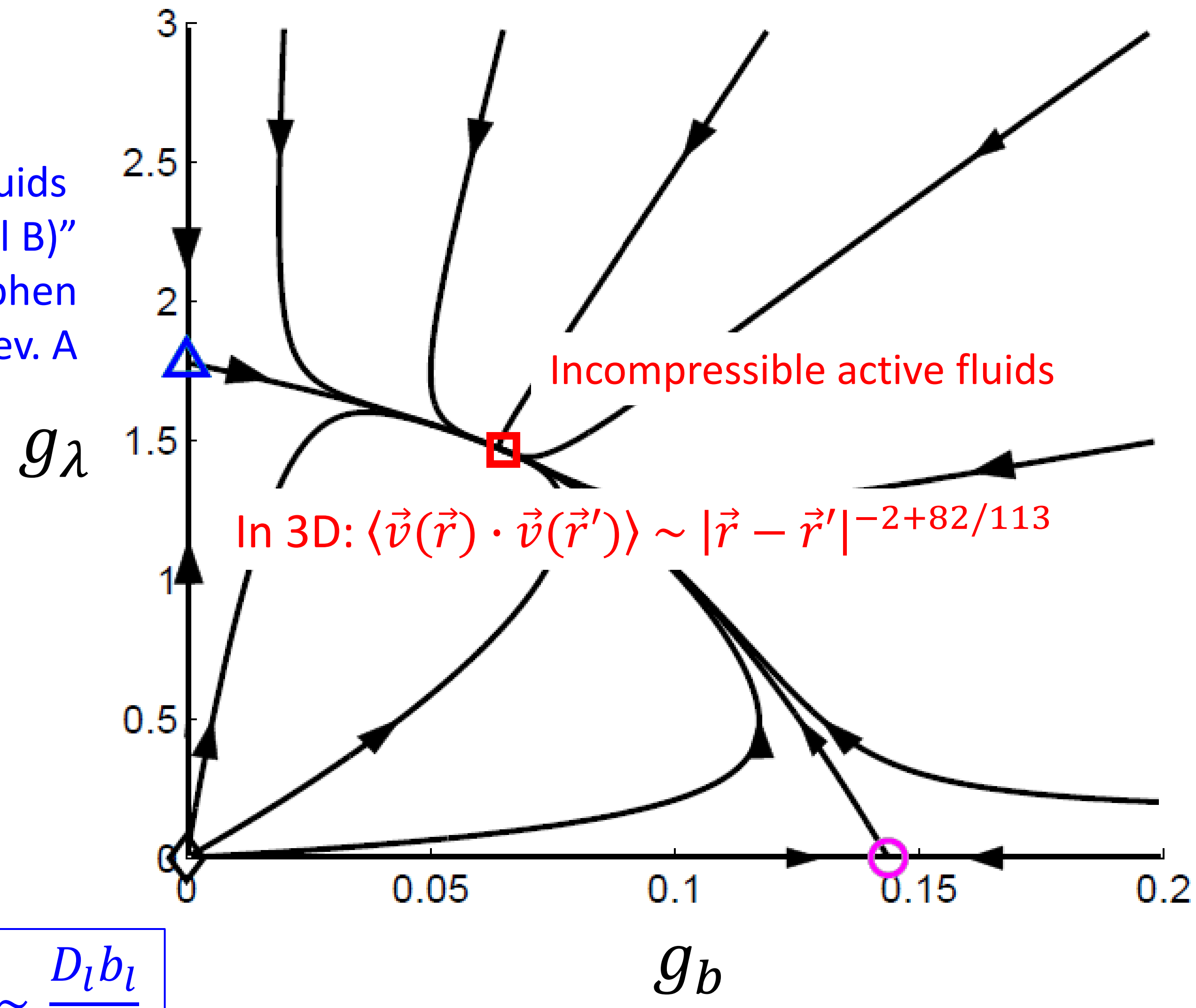

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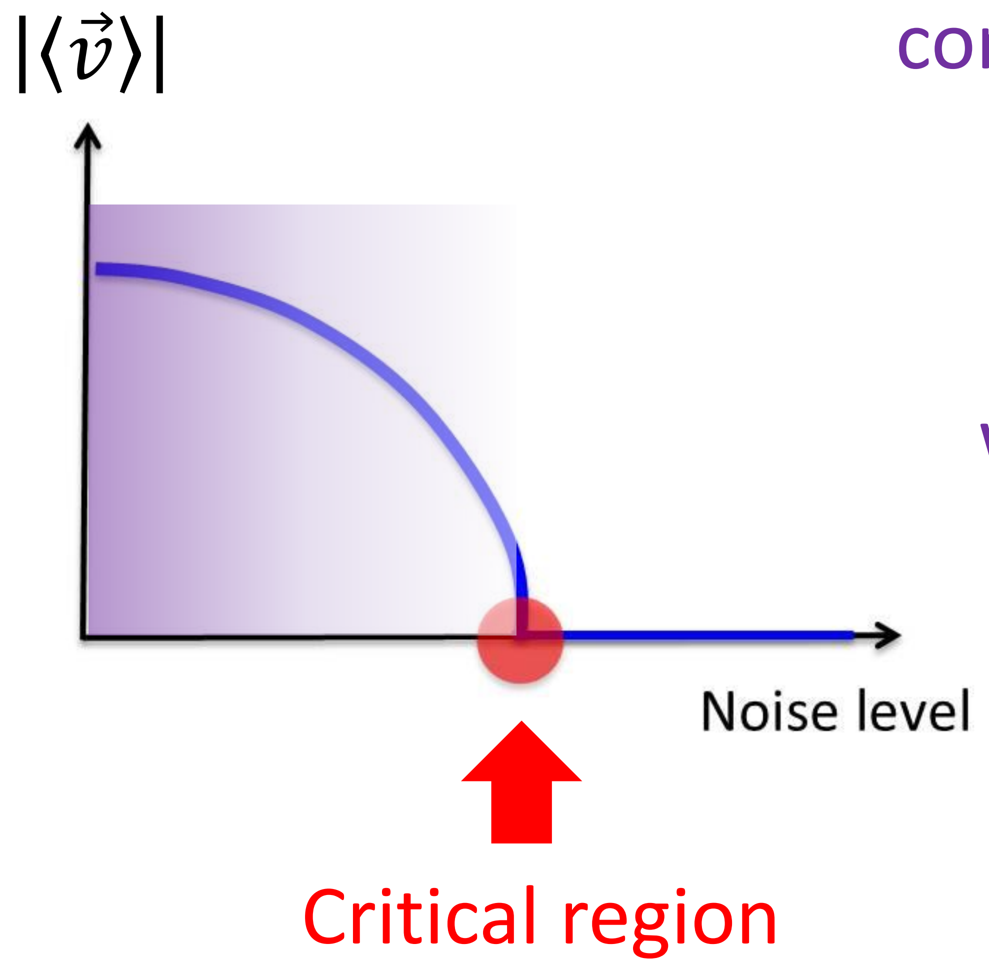
Aharony and Fisher (1973) Phys. Rev. Lett.

L Chen, J Toner, CFL (2015)
New J. Phys. 17, 042002

Ordered phase



Spontaneous breaking of a continuous (rotational) symmetry

↓

Goldstone modes (GM)

↓

What UC governs these GM?

RG analysis of the Goldstone-mode EOM in 3D:

$$\partial_t \vec{v}_\perp + \vec{\nabla}_\perp P + \vec{f}_\perp = -\lambda_l (\vec{v}_\perp \cdot \vec{\nabla}_\perp)\vec{v}_\perp - b_l v^2 \vec{v}_\perp - \mu_l \nabla^2 \vec{v}_\perp$$

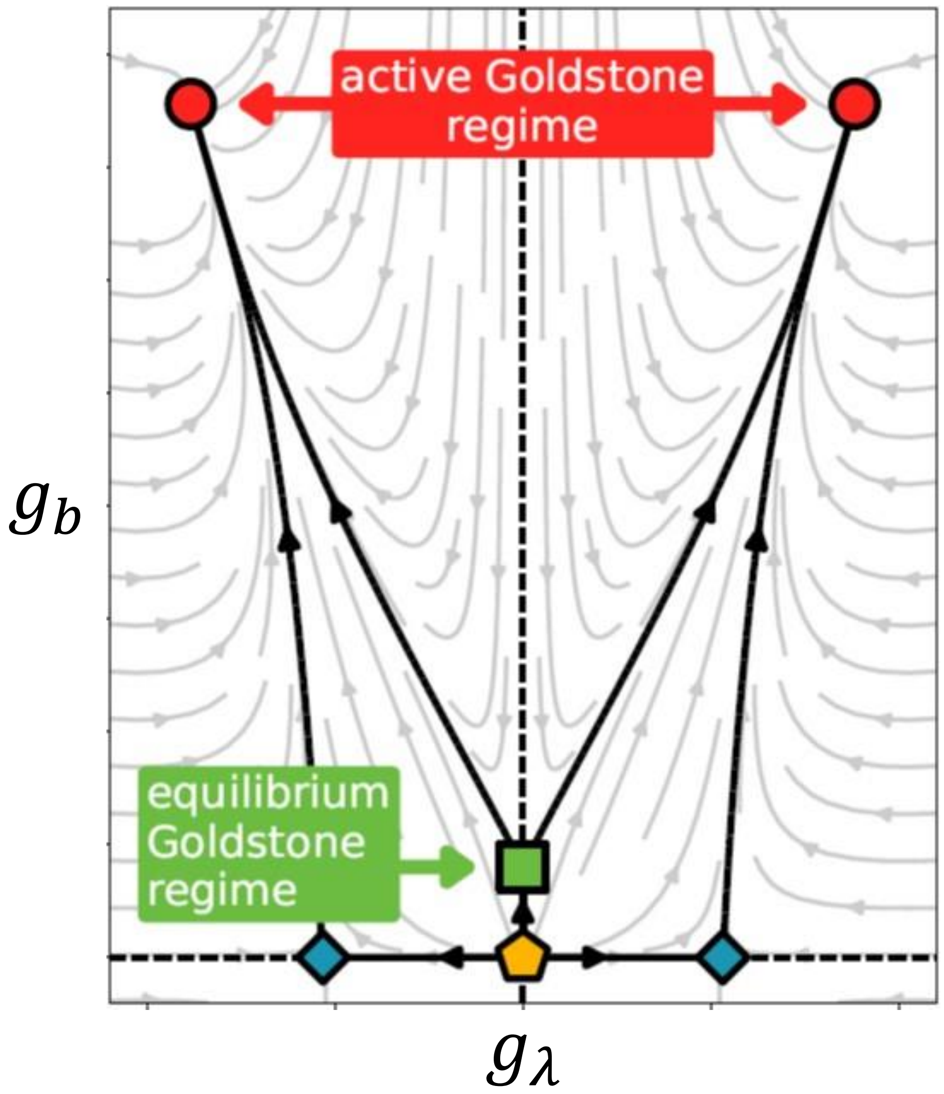
Two nonlinearities: λ_l and b_l, $l \sim$ level of RG transformation

Methodology:

Dynamic RG [L Chen, J Toner, CFL (2018) New J. Phys. 20,113035]

Functional RG [P Jentsch, CFL (2023) arXiv:2307.06725]

RG flow



L Chen, J Toner, CFL (2018)
New J. Phys. 20,113035

↓

Same UC as found in Toner & Tu (1995) PRL

P Jentsch, CFL (2023) *Can exact scaling exponents be obtained using the renormalization group? Affirmative evidence from incompressible polar active fluids*
arXiv:2307.06725

RG analysis of the EOM in 2D:

$$\partial_t \vec{v} + \vec{\nabla} P + \overrightarrow{f_l} = -(a_l + b_l v^2)\vec{v} - \mu_l \nabla^2 \vec{v}$$

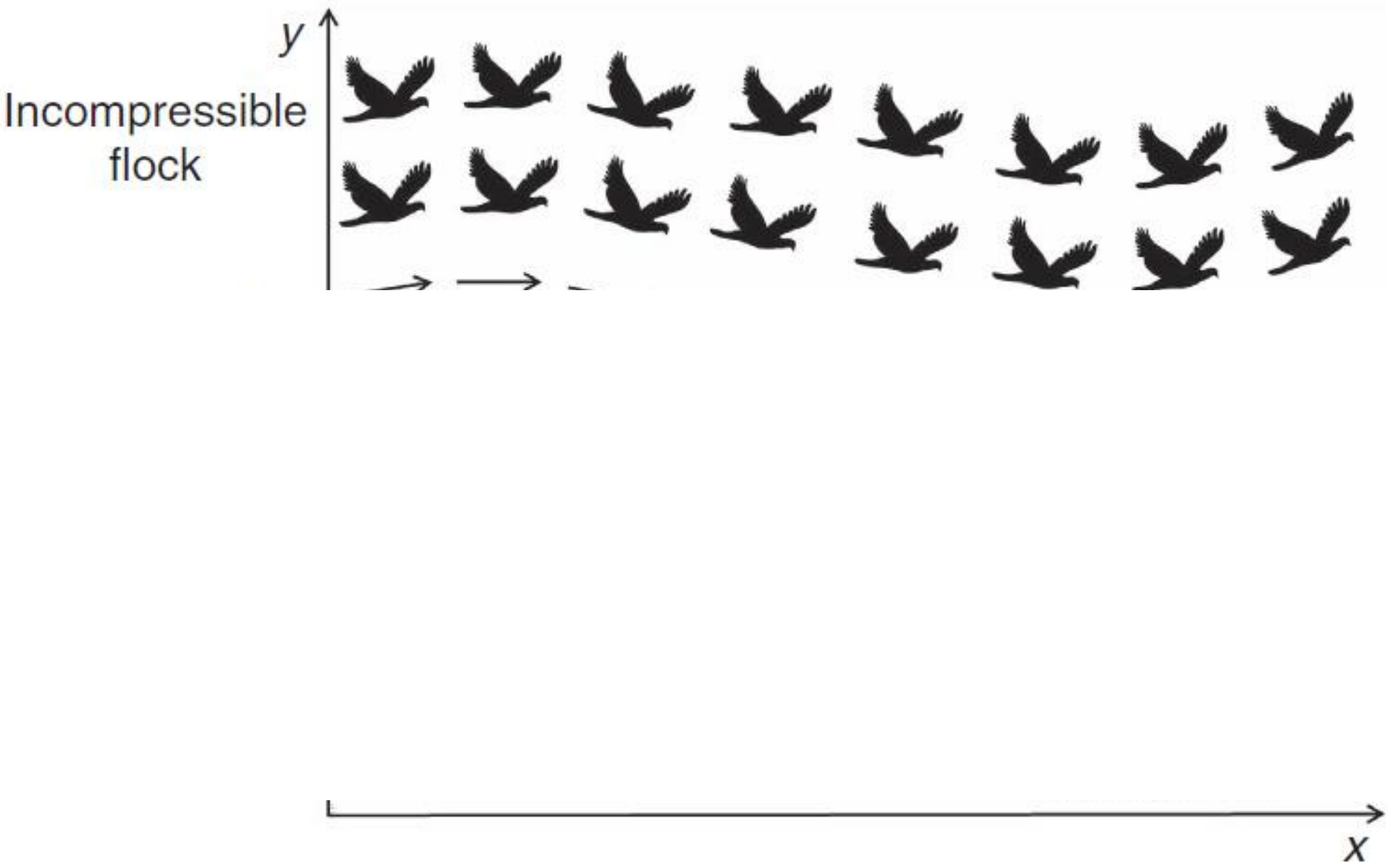
One nonlinearity: b_l, l ~ level of RG transformation

Methodology:

Dynamic RG [L Chen, CFL, J Toner (2016) Nature Communication]

Dynamic RG + ϵ-expansion at 1-loop [L. Chen, CFL, A. Maitra, J. Toner (2023) arxiv:2304.06139]

Mapping onto (1+1)-d Kardar-Parisi-Zhang



L Chen, CFL, J Toner (2016) *Surprising mappings of 2D polar active fluids to 2D soap and 1D sandblasting*
Nature Communications 7, 12215. E

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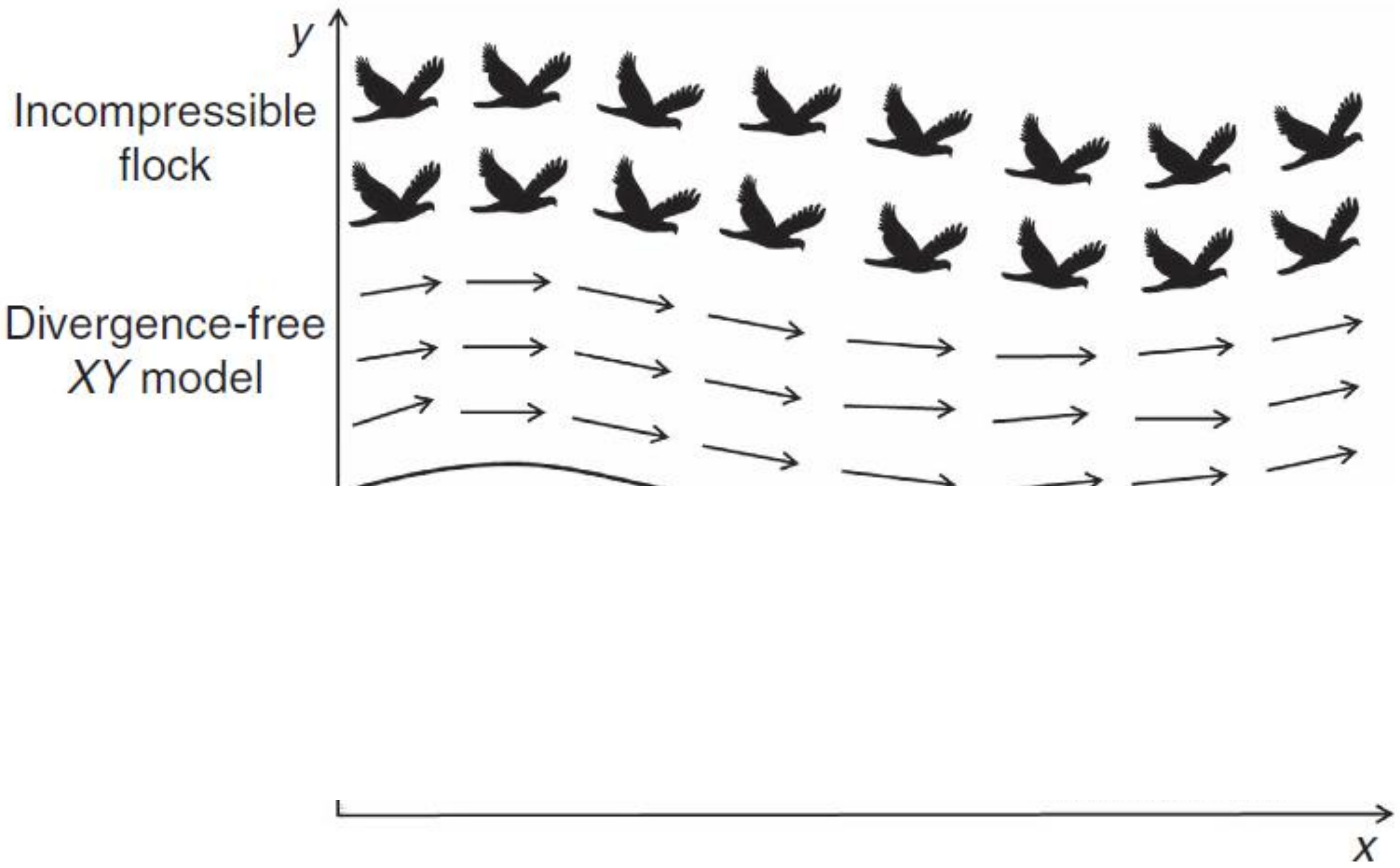

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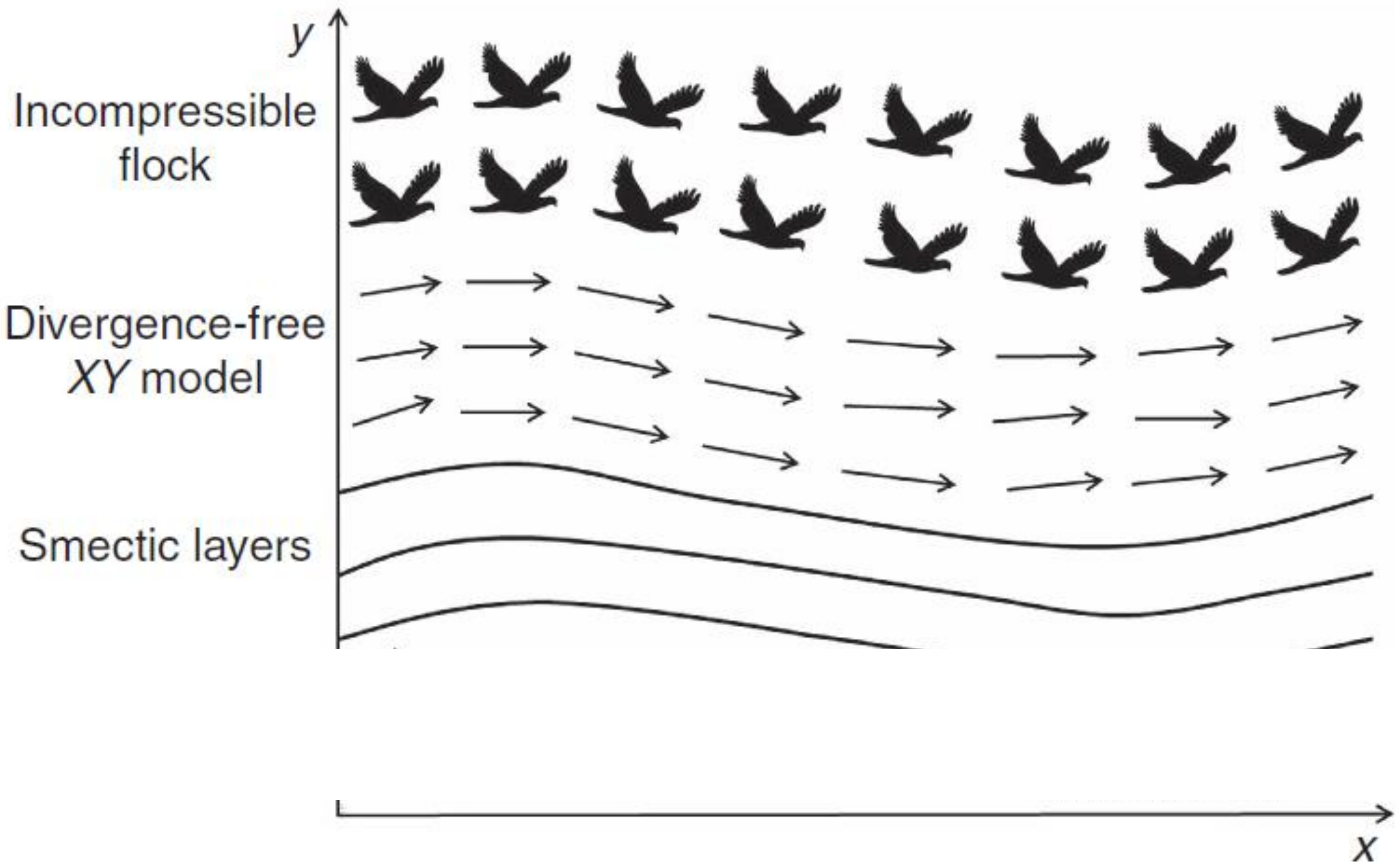

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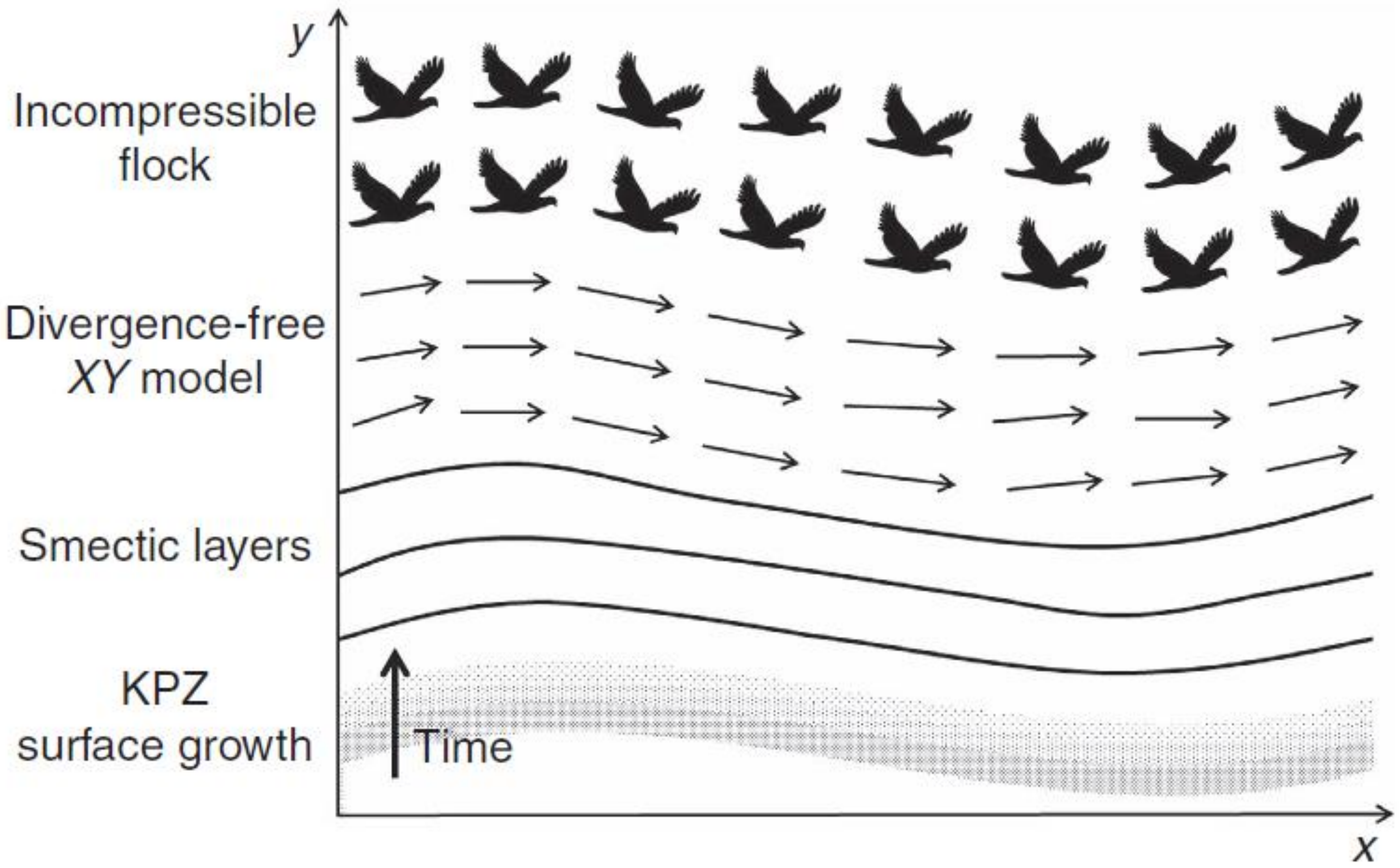
Mapping onto (1+1)-d Kardar-Parisi-Zhang



Use ϵ-expansion at 1-loop to get dynamic exponent

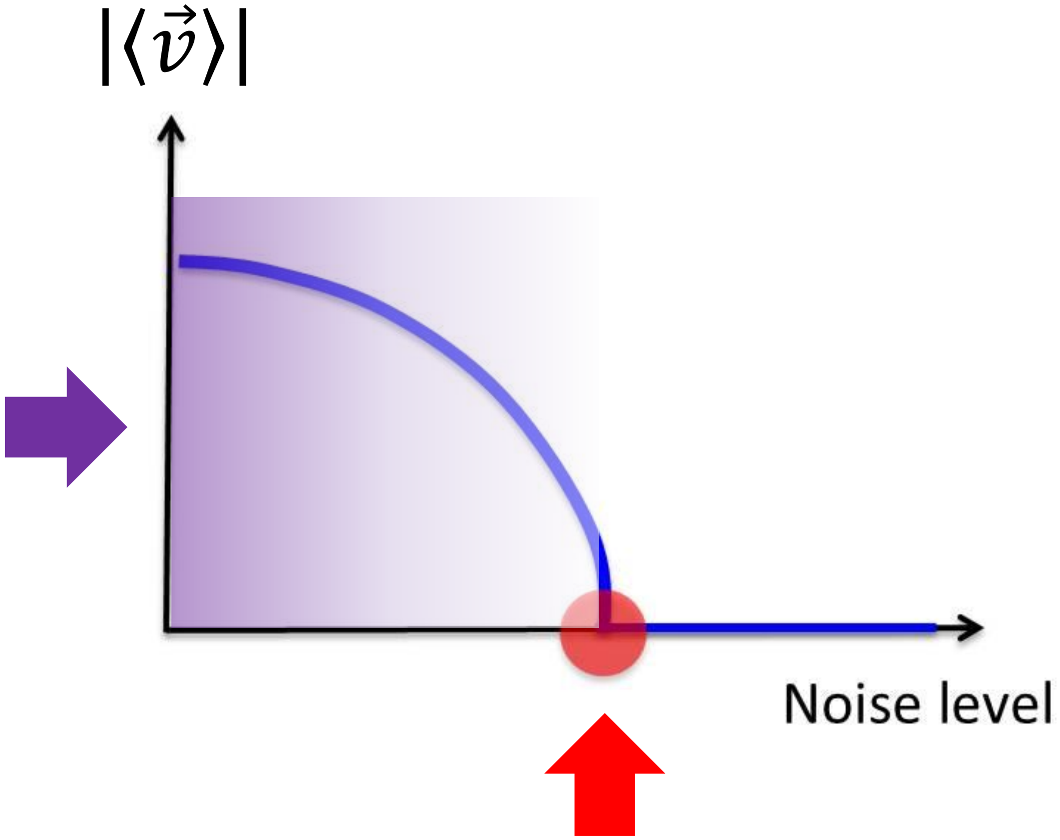
[L. Chen, C.F. Lee, A. Maitra and J. Toner (2023) *Dynamics of packed swarms: time-displaced correlators of two dimensional incompressible flocks,* arxiv:2304.06139]

→ Exact static exponents

L Chen, CFL, J Toner (2016) *Surprising mappings of 2D polar active fluids to 2D soap and 1D sandblasting* Nature Communications 7, 12215

Universal behaviour of Incomp polar active fluids

- 3D: ‘Exact’ exponents by DRG & FRG, same UC as found in Toner-Tu (1995) PRL [Chen, Lee, Toner (2018) NJP; Jentsch, Lee (2023) arxiv]
- 2D: Exact static exponents by DRG → KPZ UC [Chen, Lee, Toner (2016) Nat Comm]; **New** dynamical UC by DRG at 1-loop [Chen, Lee, Maitra, Toner (2023) arxiv]



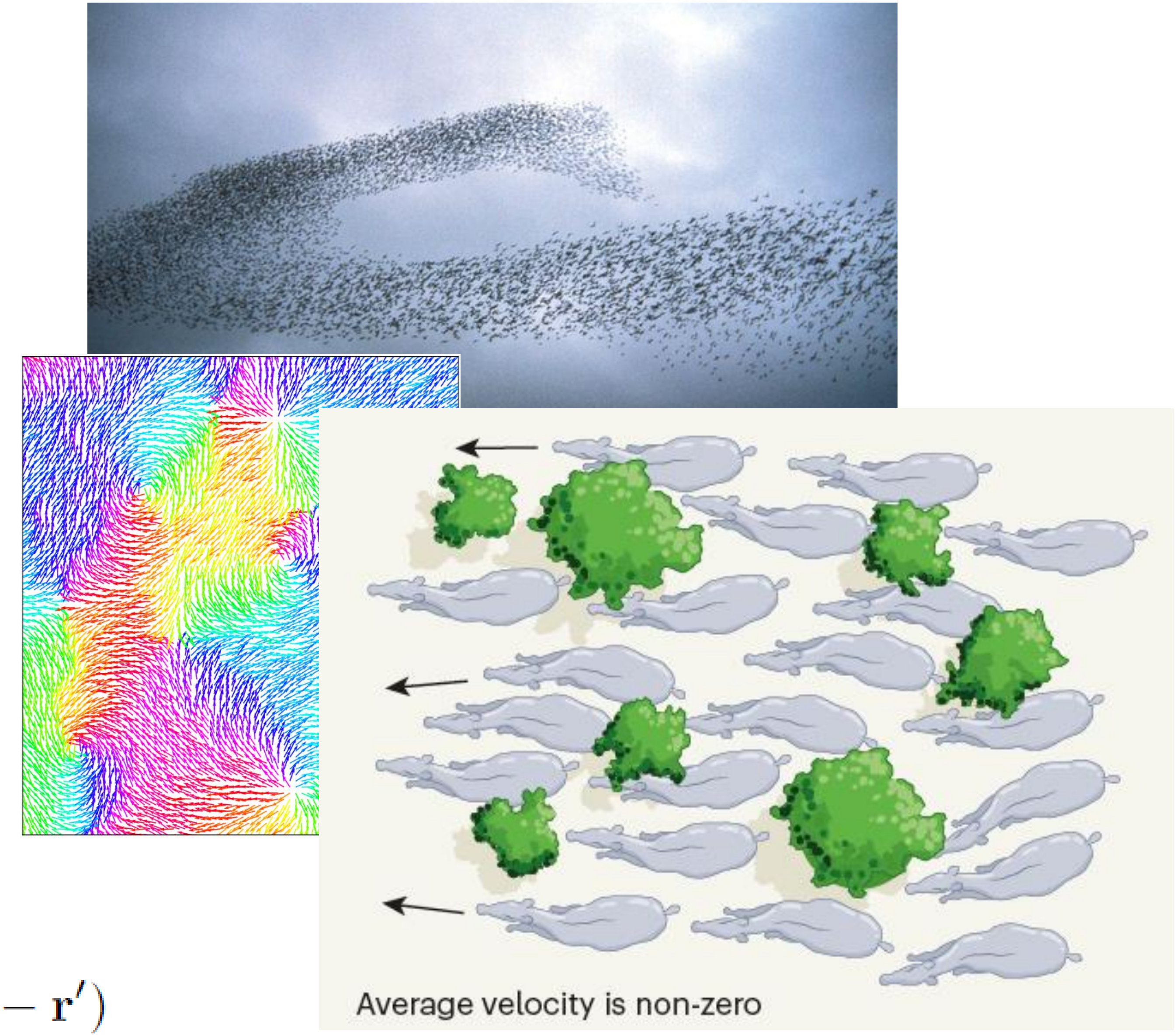
New UC: exponents by DRG at 1-loop [Chen, Toner, Lee (2015) NJP]

4. Beyond IPAF

Going beyond

- Compressible PAF
  - $\partial_t \rho = -\nabla \cdot \mathbf{j}$
- Infinitely compressible / Malthusian PAF
  - $\nabla \cdot \mathbf{v} \neq 0$
- Quenched disorder
  - Noise statistics:

$$\langle f_Q^i(\mathbf{r}, t) f_Q^j(\mathbf{r}', t') \rangle = 2D_Q \delta_{ij} \delta^d(\mathbf{r} - \mathbf{r}')$$

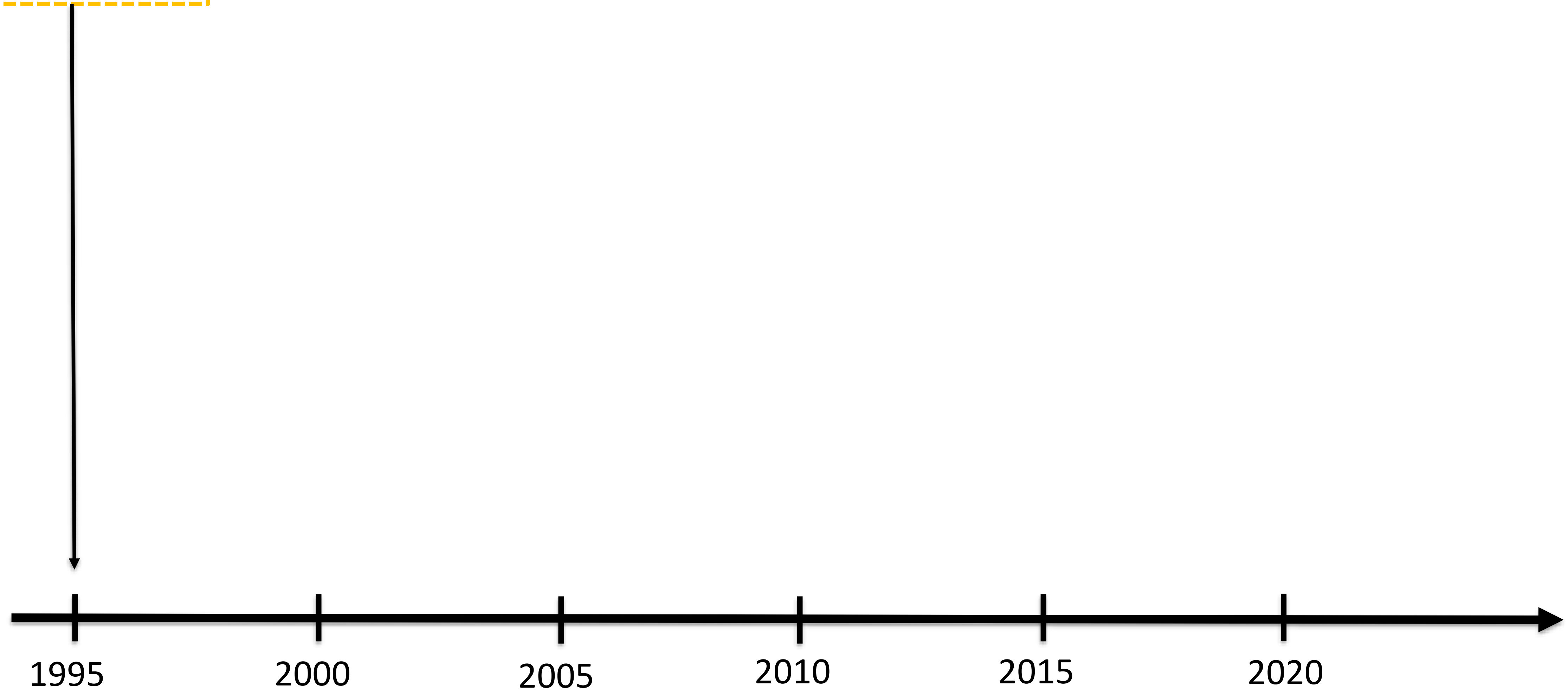


Cameron & Liverpool (2022) Nature

A treasure trove of UCs

Compressible PAF
Incomp PAF
Malthusian PAF

Toner-Tu “flocking” UC
Toner & Tu (1995) PRL



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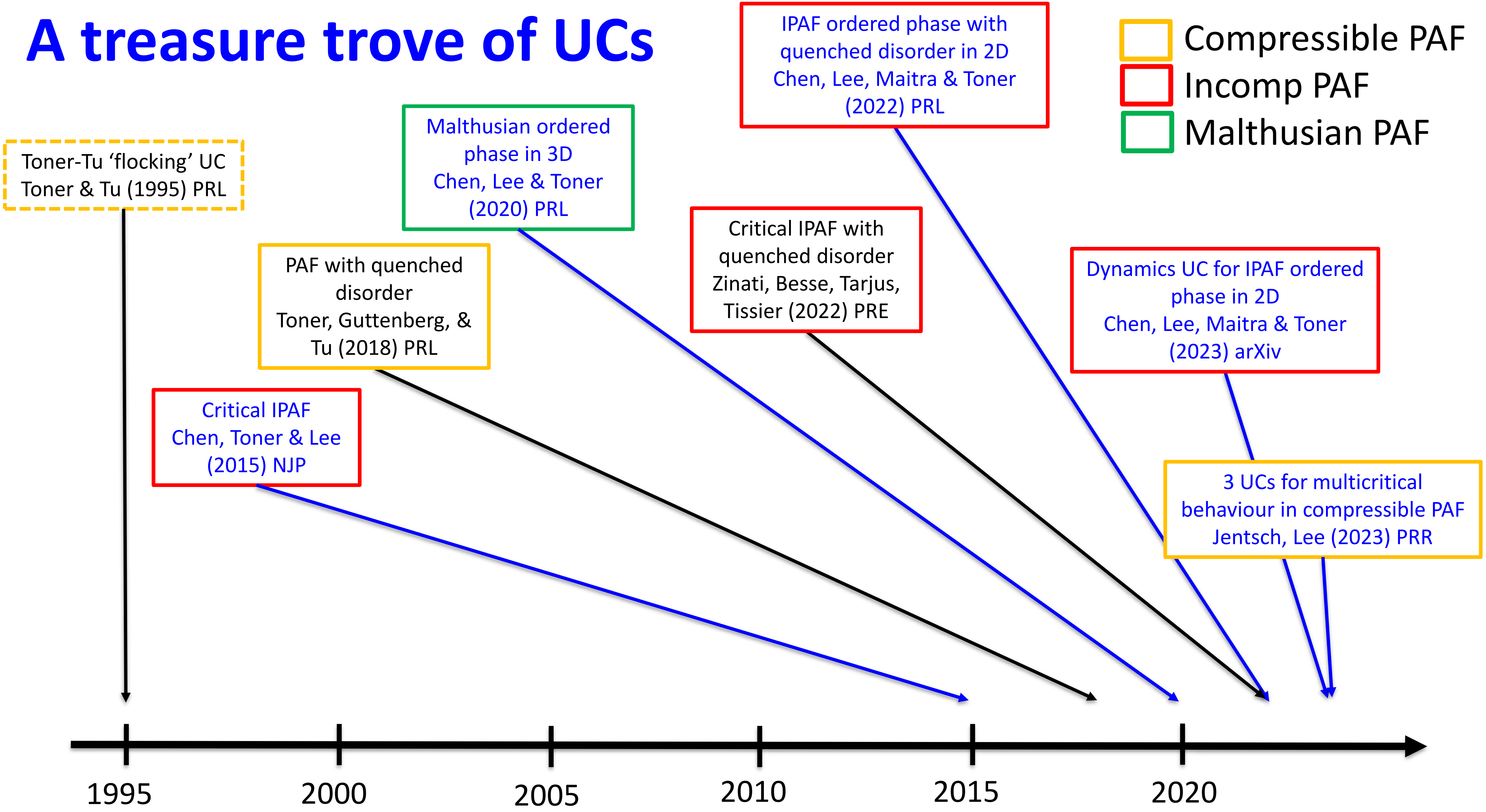
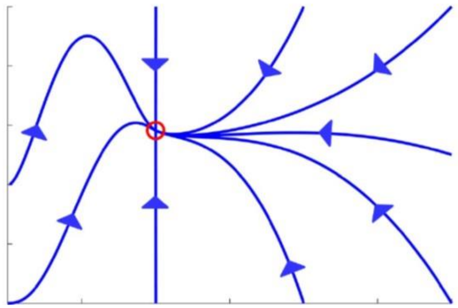
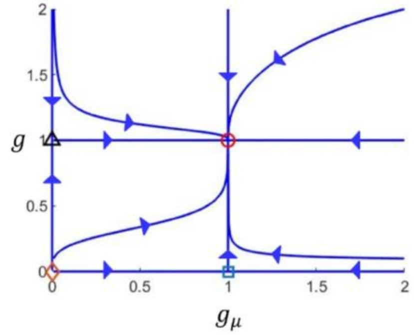
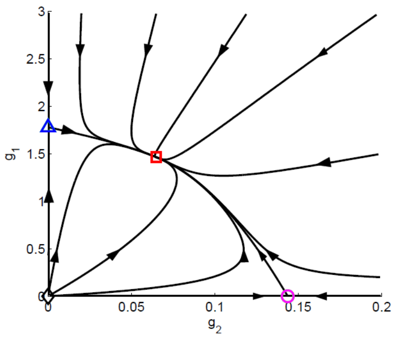
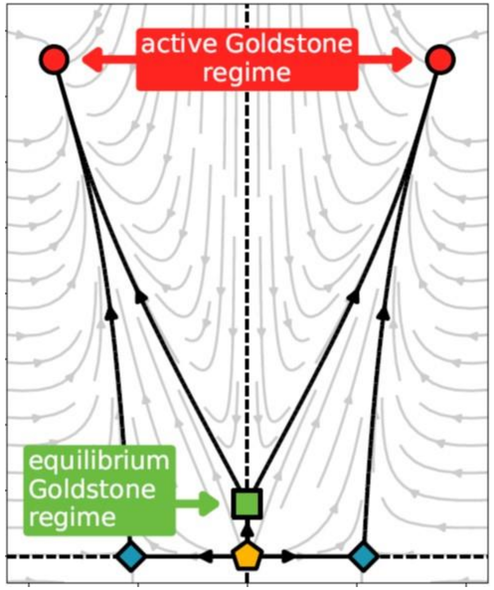


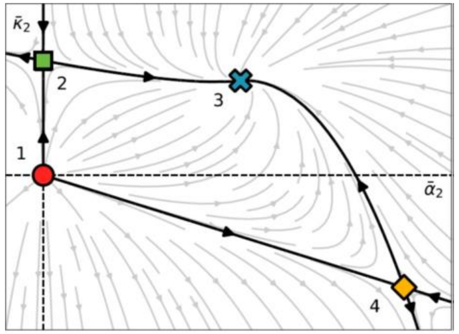






5. Summary & Outlook





Summary

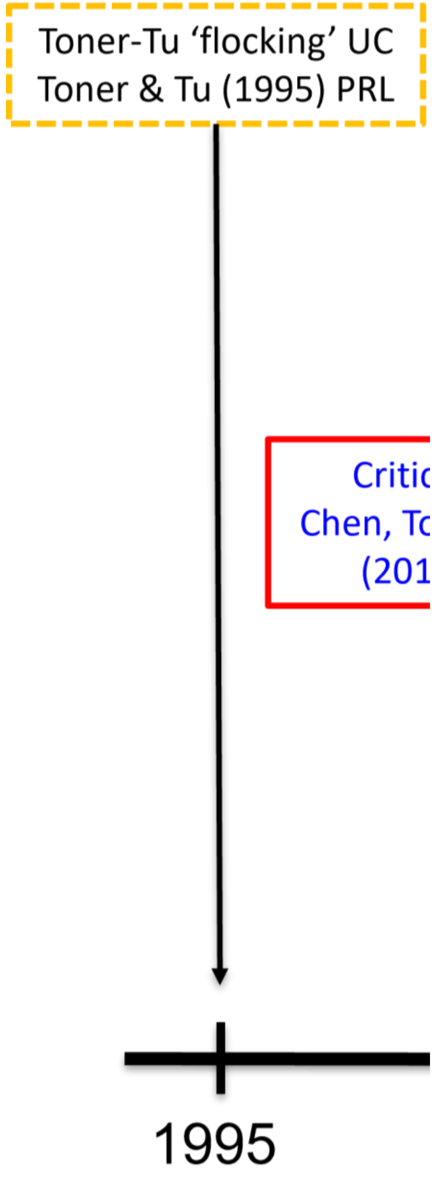
1. Universality classes (UCs) for dynamical systems ≈ periodic table for elements

2. Symmetry + conservative laws → universal EOM → RG analyses → UC identification

3. Many new UCs in polar active fluids
   - Incompressible, compressible, Malthusian

Outlook 1: Key open question

- Toner-Tu 1995 UC was supposed to describe ordered phase of generic compressible PAFs [Toner, Tu (1995) PRL]
- But Toner later realised nonlinearities missed in original 1995 analysis [Toner (2012) PRE]
- And recent simulation suggests flocking ≠ Toner-Tu 1995 UC [Mahault, Ginelli, Chaté (2019) PRL]
- *So, what is the correct UC for CPAF?*



Outlook 2

5 decades ago, technological relevance, experimental advances, and abundance of novel physics propelled condensed matter physics to become the ‘King of Physics’
[Martin (2019) Physics Today]

With its relevance to health and life, experimental advances, and abundance of novel physics, I believe biophysics will be the next ‘condensed matter physics’