

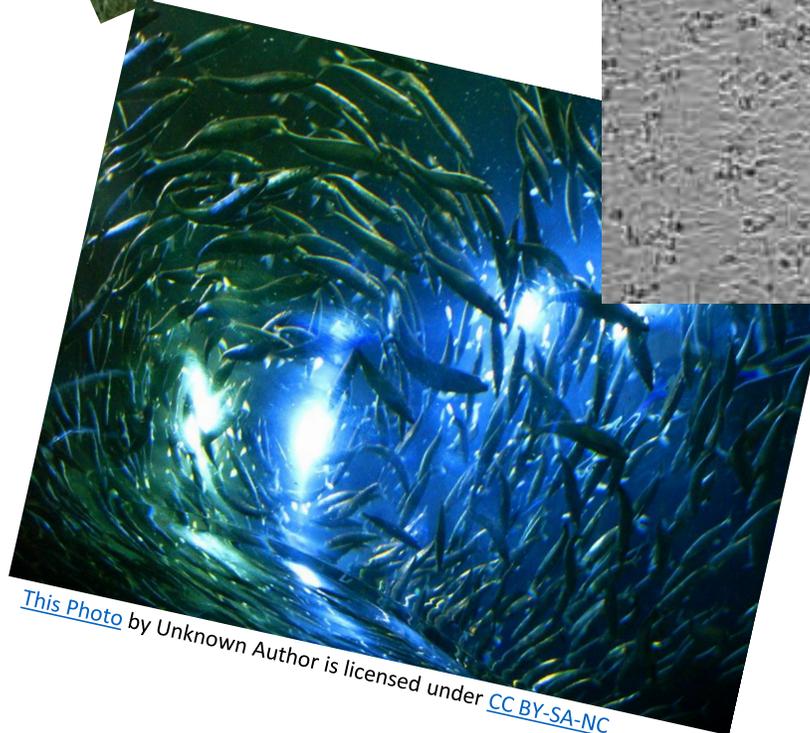
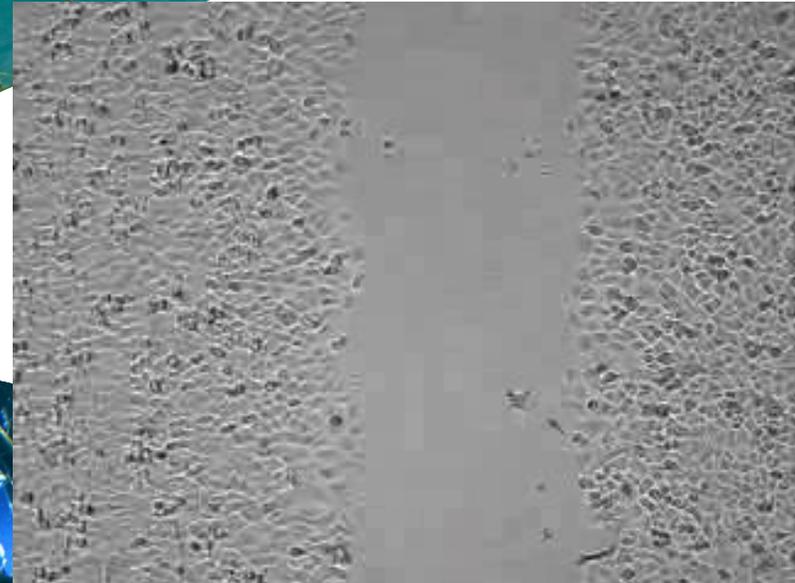
# Active Matter

## A treasure trove of novel universality classes

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[https://www.youtube.com/watch?v=v9xq\\_GiRXeE](https://www.youtube.com/watch?v=v9xq_GiRXeE)

Active matter = A many-body system in which the (microscopic) constituents can exert forces  
→ an extreme form of non-equilibrium system !

### After this talk, you will know

1. How to study the hydrodynamic behaviour of an active matter system
2. RG analyses have uncovered many novel universality classes in active matter
3. Biophysics is one of the most exciting areas of physics

# Acknowledgement

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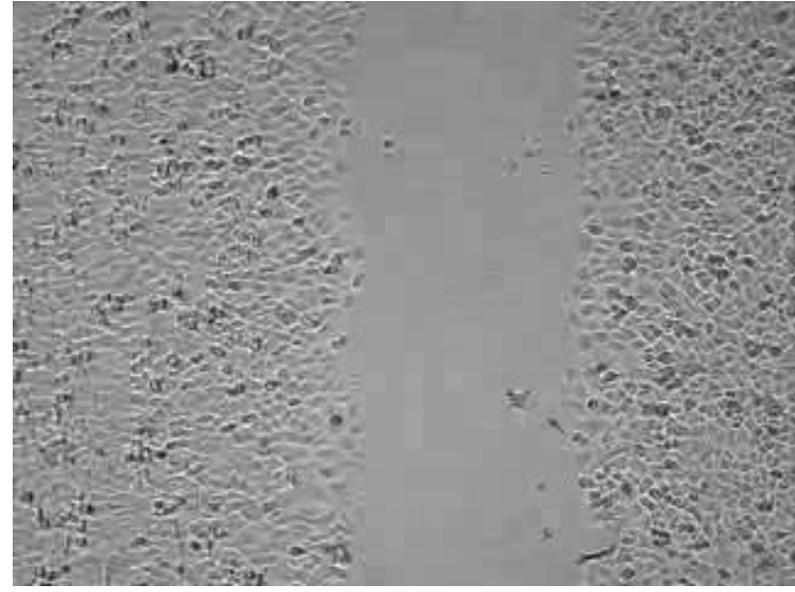


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Far from equilibrium systems:

**X** no Hamiltonian

**X** no Free energy

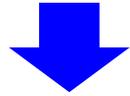
**X** no fluctuation-dissipation

✓ symmetries

✓ conservation laws

✓ constraints / exact identities

# Hydrodynamics of incompressible active fluids



Long-time, large-distance behaviour



Only 1 field (hydro. variable) to worry about: velocity  $\vec{v}$  ( $d$ -dim., vectorial)

Start with the equation of motion (EOM):  $\partial_t \vec{v} = \vec{F}$

What is  $\vec{F}$ ? Completely arbitrary except for its respect of

- 1) Symmetries: translational, rotational, temporal, chiral
- 2) Conservation law: mass conservation
- 3) Exact identity: incompressibility  $\rightarrow \nabla \cdot \vec{v} = 0$

# Hydrodynamic EOM of incompressible active fluids

$$\partial_t \vec{v} = -\vec{\nabla} P + \vec{f}_0 - \lambda_0 (\vec{v} \cdot \vec{\nabla}) \vec{v} - (a_0 + b_0 v^2) \vec{v} - \mu_0 \nabla^2 \vec{v} + c_0 v^4 \vec{v} + \xi_0 (\nabla^2)^2 \vec{v} + \dots$$



Gaussian noise

Higher order terms in  $\nabla$  &  $\vec{v}$

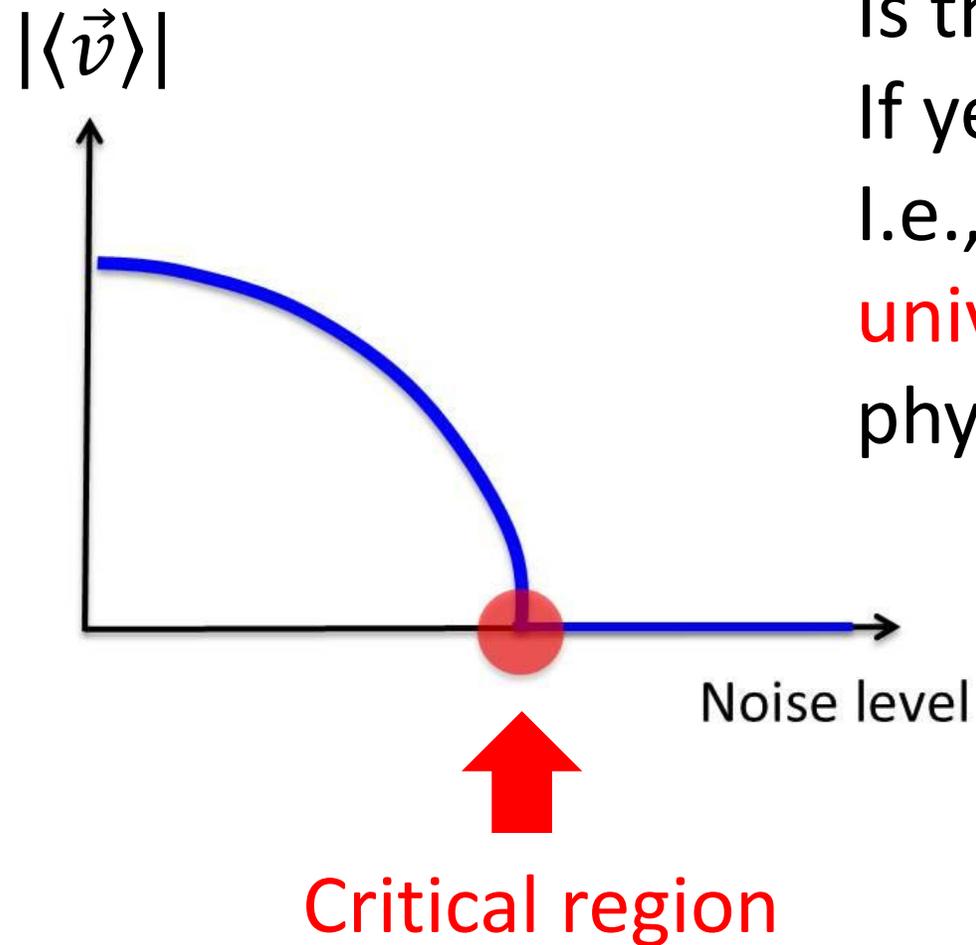
'pressure' (Lagrange multiplier)  
to enforce incompressibility

Now that we have a model (a.k.a. Toner-Tu model [Toner & Tu (1995) PRL]), what do we do next?

Simplest thing first  $\rightarrow$  mean-field analysis:  $(a_0 + b_0 v^2) = 0$

$a_0 < 0 \rightarrow$  ordered phase with non-zero  $\vec{v}$  : **active** collective motion

# Order-disorder critical transition



Is the transition really **critical**?  
If yes, is it a **new** critical phenomenon?  
I.e., does it correspond to a **new**  
**universality class** in non-equilibrium  
physics?

RG analysis of the EOM:

$$\partial_t \vec{v} + \vec{\nabla} P + \vec{f}_l = -\lambda_l (\vec{v} \cdot \vec{\nabla}) \vec{v} - (a_l + b_l v^2) \vec{v} - \mu_l \nabla^2 \vec{v}$$

Two nonlinearities:  $\lambda_l$  and  $b_l$ ,  $l \sim$  level of RG transformation

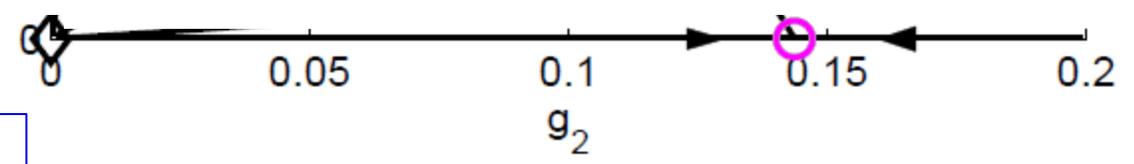
Methodology:

Dynamic renormalization group +  $\epsilon$ -expansion at 1-loop

[L Chen, J Toner, CFL (2015) New J. Phys. 17, 042002]

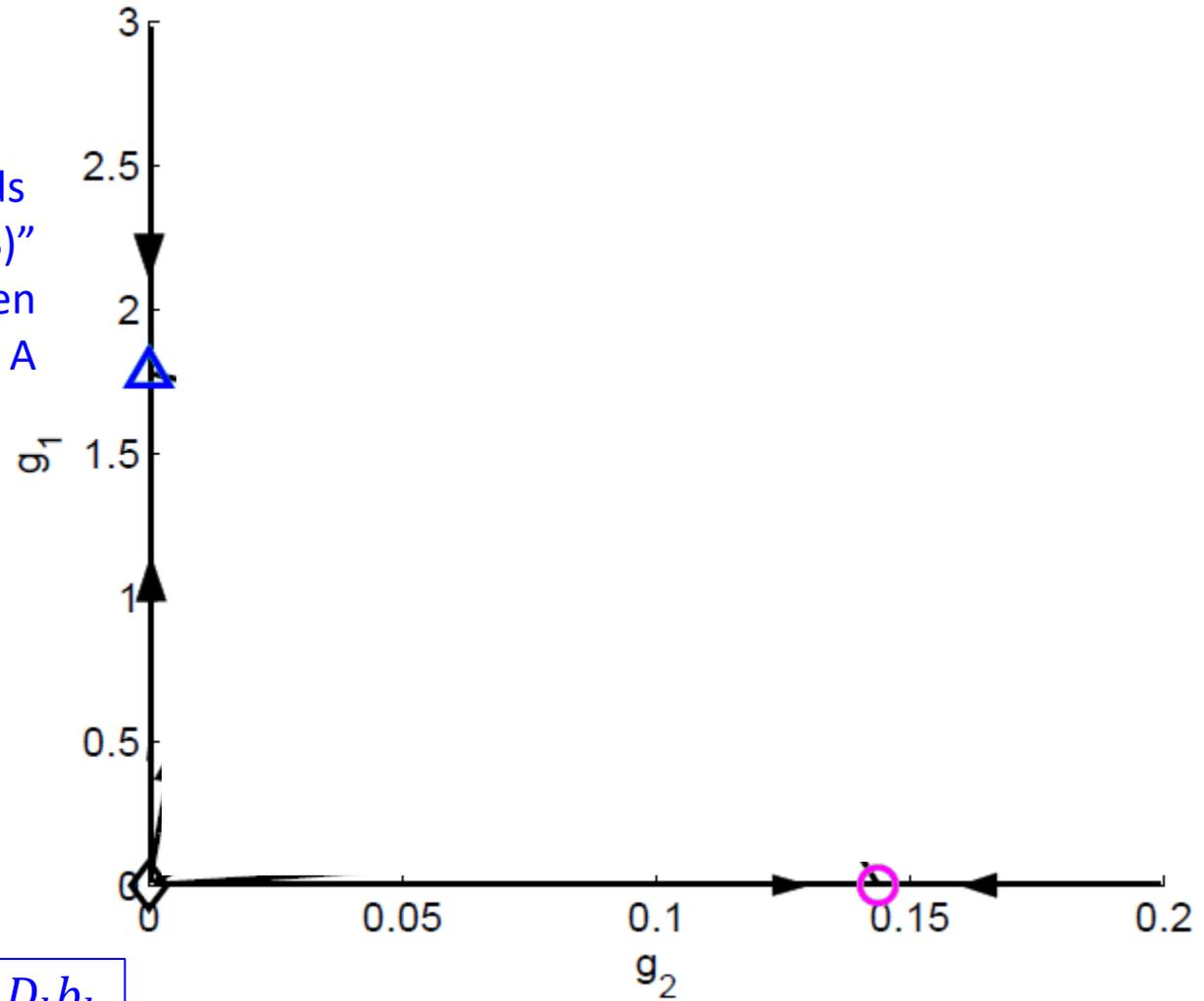
$$g_1(l) \sim \frac{D_l \lambda_l^2}{\mu_l^3}; \quad g_2(l) \sim \frac{D_l b_l}{\mu_l^2}$$

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New J. Phys. 17, 042002



“Ferromagnets with dipolar interactions”  
Aharony and Fisher (1973) Phys. Rev. Lett.

“Randomly stirred fluids  
(Model B)”  
Forster, Nelson & Stephen  
(1977) Phys. Rev. A

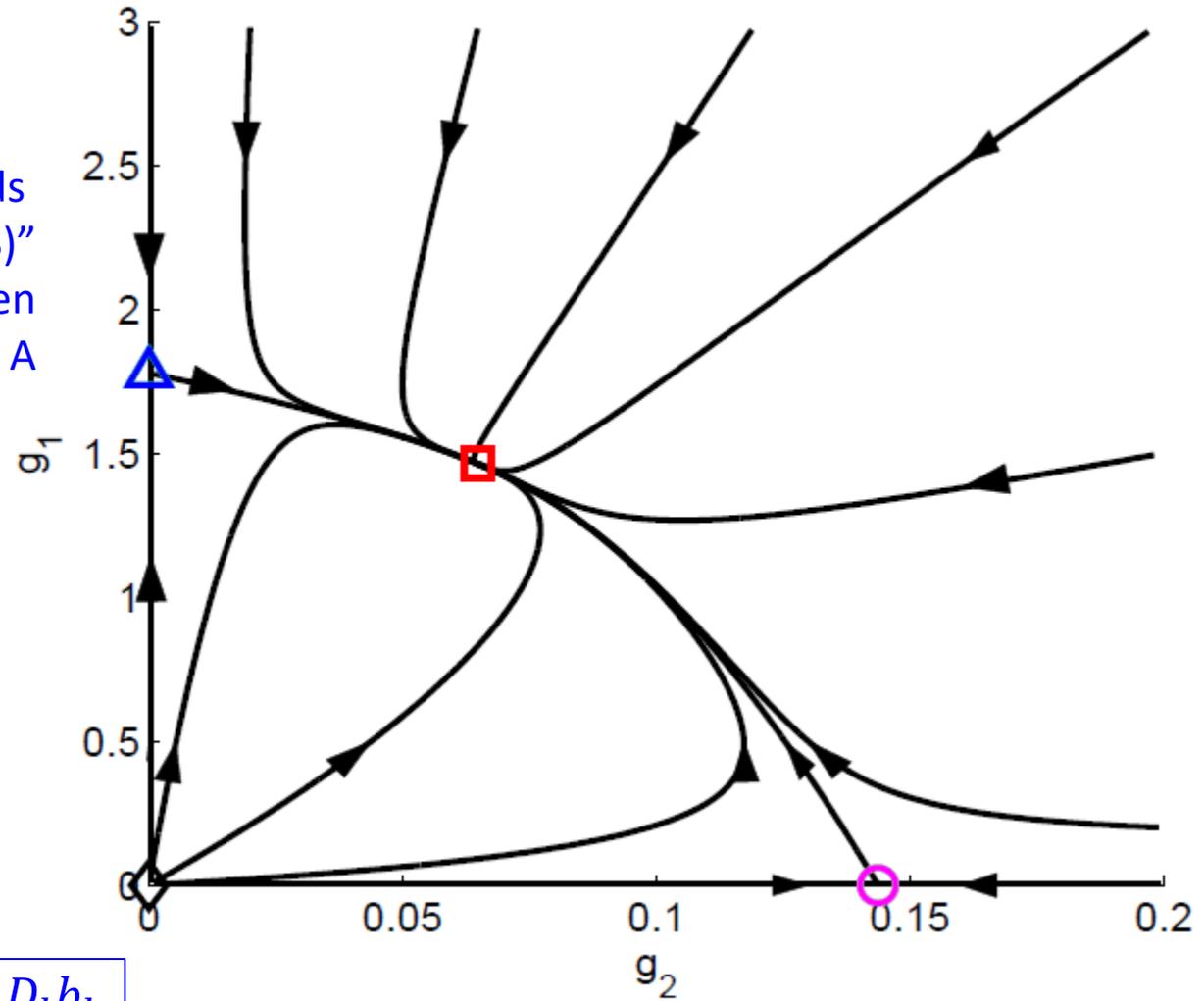


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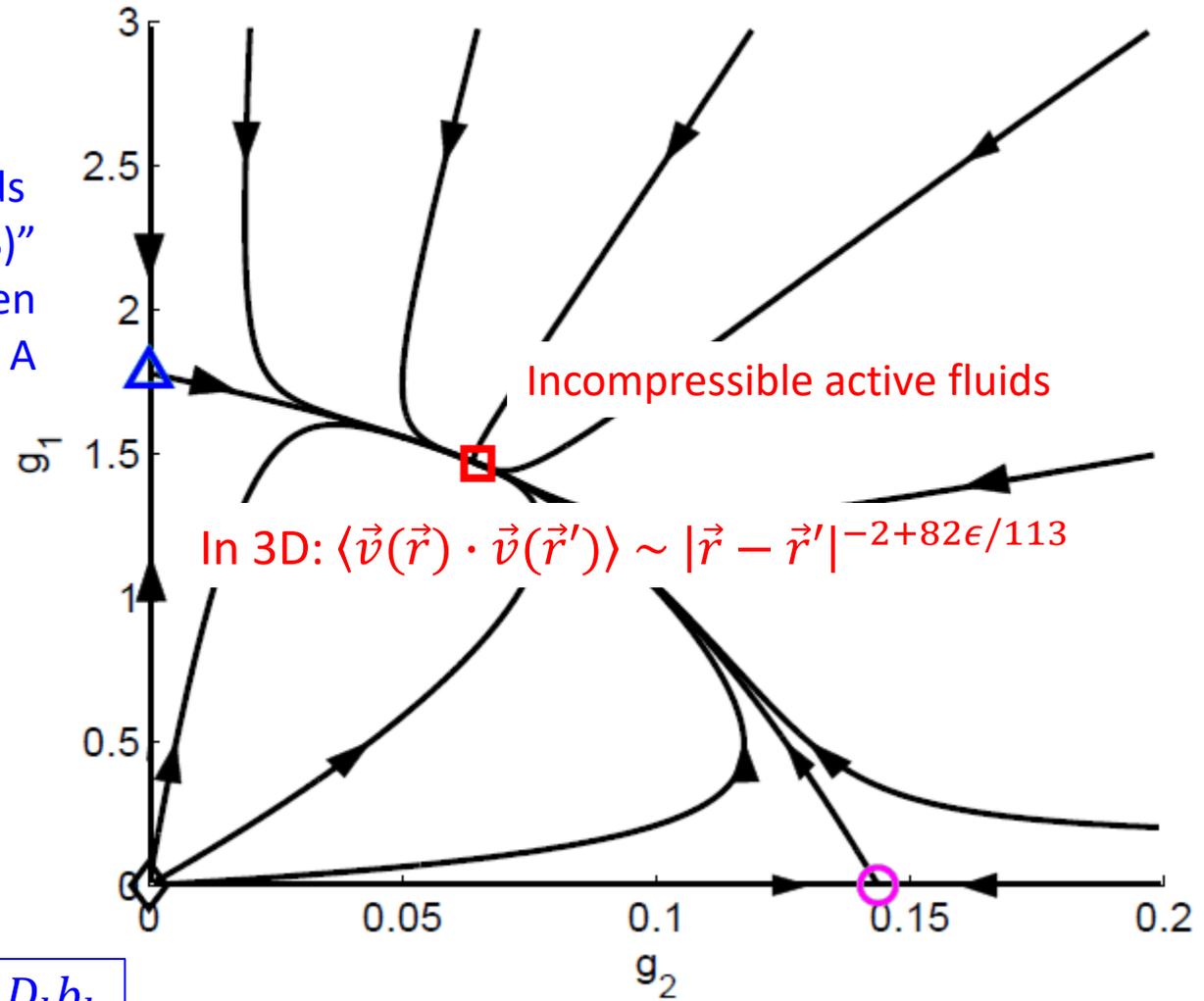


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# A treasure trove of novel universality classes (UC)

## New order-disorder critical phenomena

- Incomp. active fluids (IAF) [Chen, Toner, CFL (2015) NJP]
- IAF with **quenched random field disorder** [Zinati, Besse, Tarjus, Tissier (2022) PRE]
- **Multicritical phenomena** in **compressible** active fluids – see next talk [Jentsch, CFL, arXiv:2205.01610]

## New ordered phases

- IAF for  $2 < d < 4$  [Toner & Tu (1995) PRL; Chen, CFL, Toner (2018) NJP]

**Surprise:  $2d$  IAF belongs to the  $(1+1)d$  Kardar-Parisi-Zhang UC** [Chen, CFL, Toner (2016) Nature Comm]

- IAF with **quenched random field disorder** for  $2 \leq d < 5$  [Chen, CFL, Maitra, Toner, arXiv:2202.02865 & arXiv:2203.01892]
- **Infinitely compressible** active fluids for  $2 \leq d < 4$  [Toner (2012) PRL; Chen, CFL, Toner (2020) PRL, PRE]

And many more new UC are expected...

In particular, a key open question is:

*What is the universal behaviour of ordered compressible active fluids?*



At the beginning, I promised that you will know by now

1. How to study the hydrodynamic behaviour of an active matter system
2. RG analyses have uncovered many novel universality classes in active matter
3. *Biophysics is one of the most exciting areas of physics*

# Vision

5 decades ago, technological relevance, experimental advances, and abundance of novel physics propelled condensed matter physics to become the 'King of Physics'

[Martin (2019) Physics Today]

With its relevance to health and life, experimental advances, and abundance of novel physics, I believe biophysics will become the next 'King of Physics'