DAMTP Cambridge, Feb 20, 2018

Universality in active matter

Chiu Fan Lee

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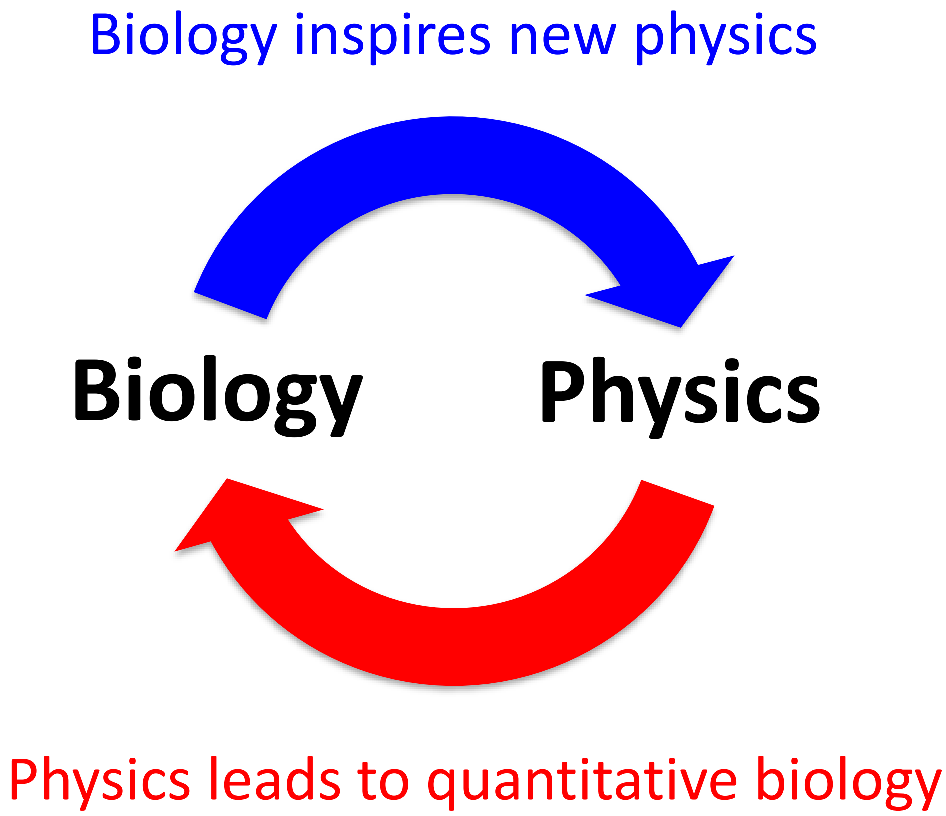
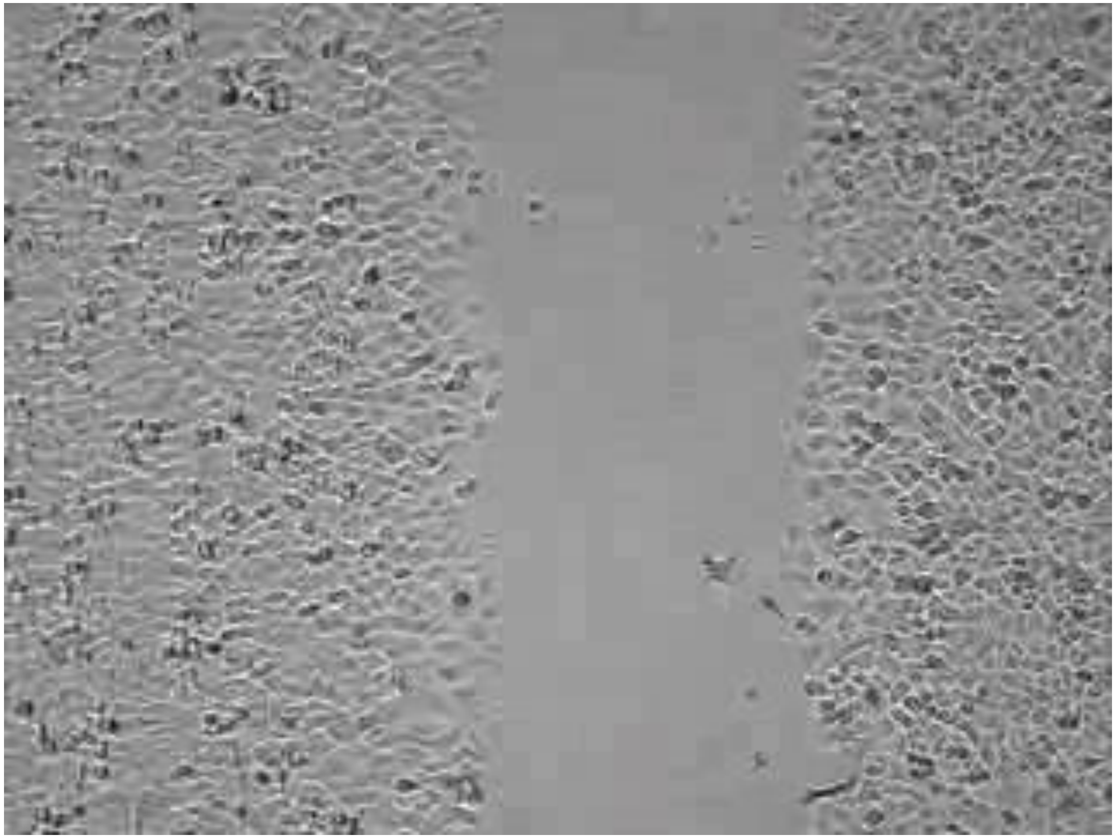

Biology inspires new physics
Biology
Physics
Physics leads to quantitative biology



https://www.youtube.com/watch?v=v9xq_GiRXeE

Contrary to Saffman-Taylor instability, there is instability, and the instability length scale is of the system size

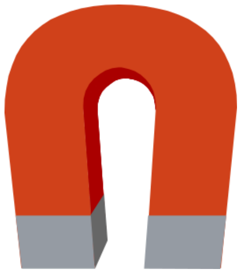
D. Nesbitt, G. Pruessner & CFL (2017)
Edge instability in incompressible planar active fluids. Phys. Rev. E

Main messages

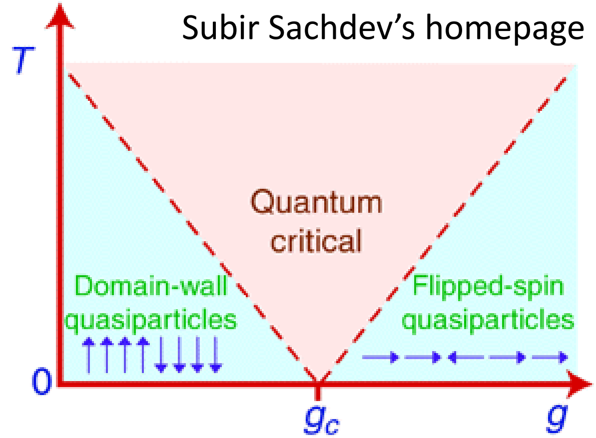
- Symmetry -> active matter
- Connections to diverse known physical systems
- Emergence of new universality classes

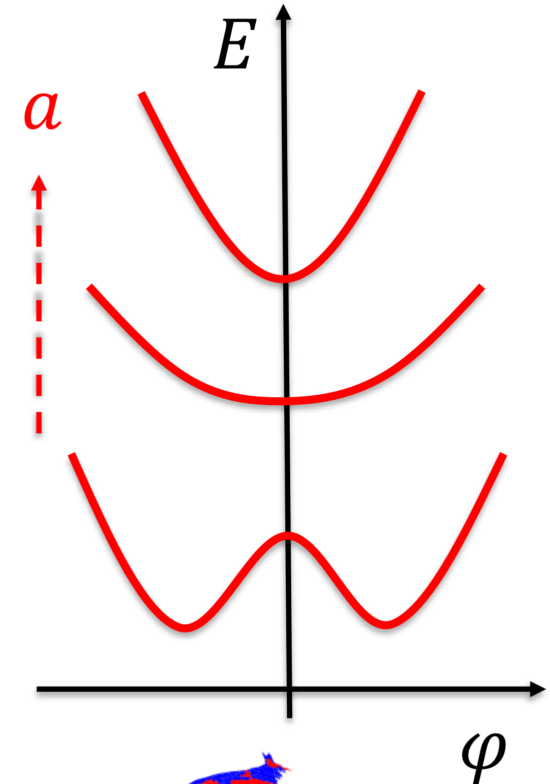
Plan

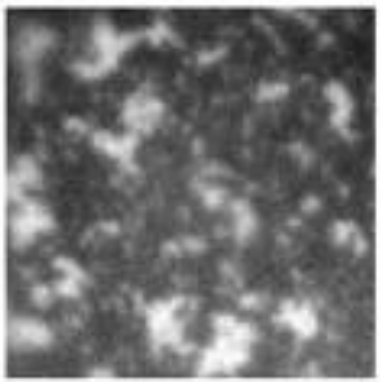
- Discrete symmetry: $\boldsymbol{Z}_2$-symmetry
  - Equilibrium Ising model
  - Active Ising model
- Continuous symmetry: SO(d)-symmetry
  - Incompressible thermal fluids (Navier-Stokes)
  - Incompressible active fluids at criticality and in the ordered phase
- Summary & Outlook



Subir Sachdev's homepage

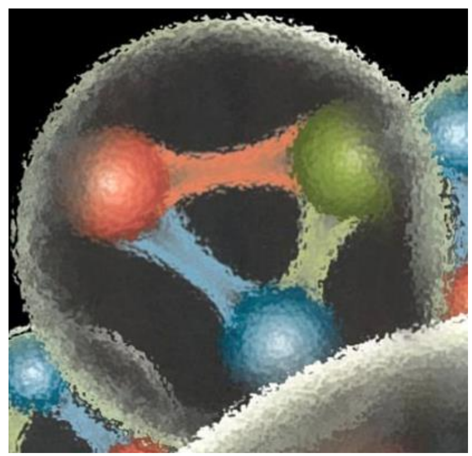




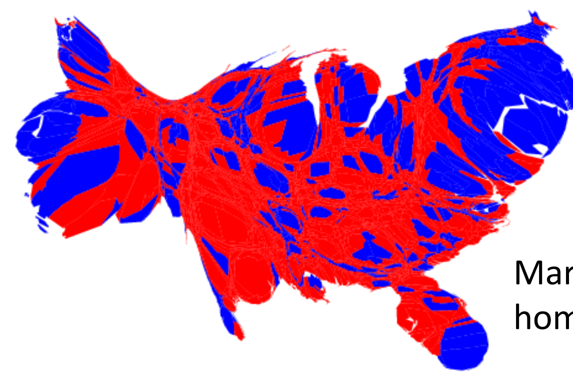


Honerkamp-Smith et al. (2009) BBA

Confinement-deconfinement transition in SU(2) gauge field theory



B. Svetitsky & L. Yaffe (1982) Nucl. Phys. B210, 423



Mark Newman's homepage

1. Discrete symmetry: Z_2-symmetry

Symmetry constrains Hamiltonian $\mathcal{H}$

- One coarse-grained field: φ
- Temporal invariance: $\mathcal{H}$ does not depend on t
- Translational inv.: $\mathcal{H}$ does not depend on $\mathbf{r}$
- Rotational inv.: Only differential terms like $|\nabla\varphi|^2$, $(\nabla^2\varphi)^2$, etc, are in $\mathcal{H}$
- In addition, we want $\boldsymbol{Z}_2$-symmetry: $\mathcal{H}[\phi] = \mathcal{H}[-\phi]$

$$\mathcal{H} = \int \mathrm{d}^d\mathbf{r}\left[\mu|\nabla\phi|^2 + a\phi^2 + b\phi^4 + \mu'\left(\nabla^2\phi\right)^2 + b'\phi^6\cdots\right]$$

Model A dynamics

- Non-conservative dynamics: $\frac{\partial \phi}{\partial t} = -\frac{\delta H}{\delta \phi} + f$

$$\langle f(\mathbf{r}, t) \rangle = 0$$

$$\langle f(\mathbf{r}, t) f(\mathbf{r}', t') \rangle = 2D\delta(\mathbf{r} - \mathbf{r}')\delta(t - t')$$

- Equation of motion (EOM):

$$\partial_t \phi = \mu \nabla^2 \phi - a\phi - b\phi^3 + f + \mu' \nabla^4 \phi + b' \phi^5 + \cdots$$

Critical (scale invariant) dynamics 1

- Start with the linearised EOM: $\partial_t \phi = \mu \nabla^2 \phi + f$
- Re-scales the coordinates and field:

$$\mathbf{r} \to \mathrm{e}^{\ell}\mathbf{r} \quad , \quad t \to \mathrm{e}^{z\ell} t \quad , \quad \phi \to \mathrm{e}^{\chi \ell}\phi$$

$$\Rightarrow \quad \mathrm{e}^{(\chi - z)\ell}\partial_t \phi = \mu \mathrm{e}^{(\chi-2)\ell}\nabla^2\phi + \mathrm{e}^{-(d+z)\ell/2} f$$

$$\partial_t \phi = \mu \mathrm{e}^{(z-2)\ell}\nabla^2\phi + \mathrm{e}^{(z-2\chi-d)\ell/2} f$$

$$\left.\begin{aligned} \frac{1}{\mu_\ell}\frac{\mathrm{d}\mu_\ell}{\mathrm{d}\ell} &= (z-2) \\ \frac{1}{D_\ell}\frac{\mathrm{d}D_\ell}{\mathrm{d}\ell} &= (z - 2\chi - d) \end{aligned}\right\} \quad z_{\mathrm{lin}} = 2 \quad , \quad \chi_{\mathrm{lin}} = -\frac{d-2}{2}$$

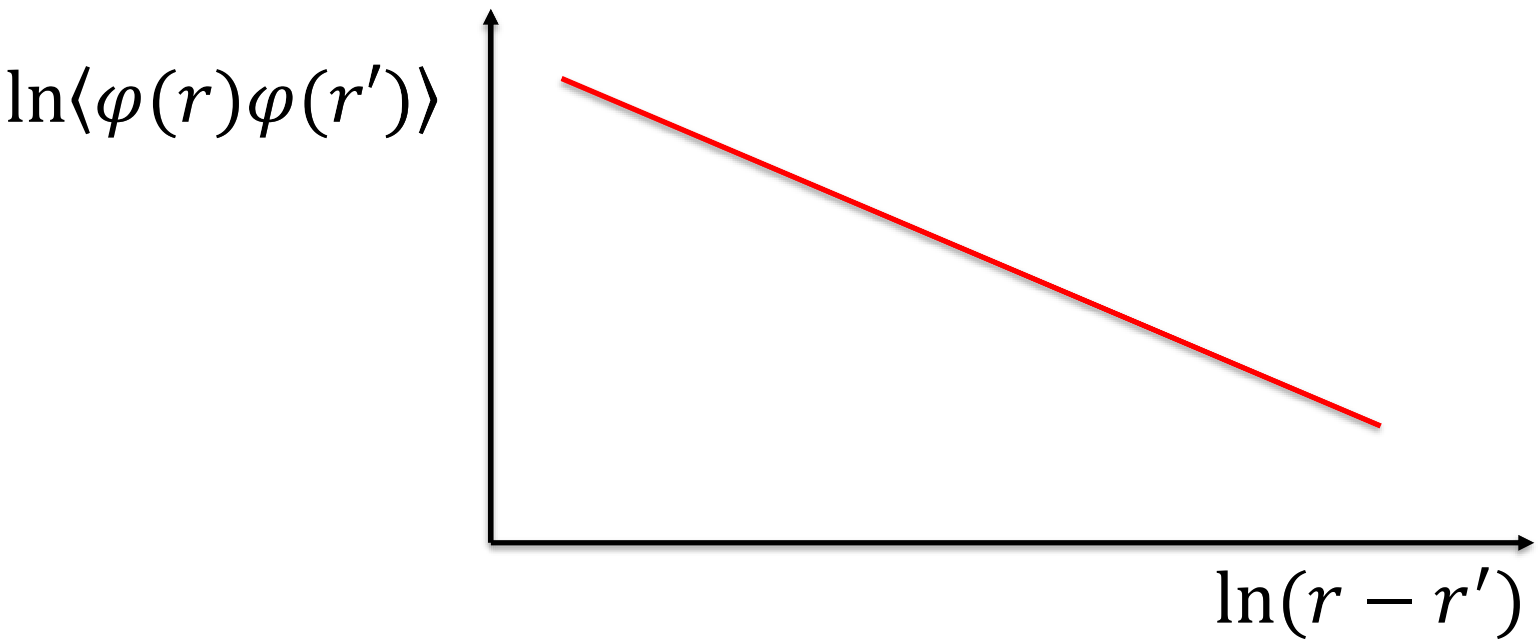
S.F. Edwards & D.R. Wilkinson (1982)
Proc. Royal Soc. London A

Scale invariant pattern

$$\mathrm{R} = \alpha \mathrm{r} \quad , \quad \mathrm{R}' = \alpha \mathrm{r}'$$

$$\alpha = \mathrm{e}^{\ell}$$

$$\langle \phi(\mathbf{R})\phi(\mathbf{R}')\rangle = \langle \phi(\mathrm{e}^{\ell}\mathbf{r})\phi(\mathrm{e}^{\ell}\mathbf{r}')\rangle = \langle \mathrm{e}^{-\chi\ell}\phi(\mathbf{r})\mathrm{e}^{-\chi\ell}\phi(\mathbf{r}')\rangle = \alpha^{-2\chi}\langle \phi(\mathbf{r})\phi(\mathbf{r}')\rangle$$



Critical (scale invariant) dynamics 2

- How about the nonlinearities?

$$\partial_t\phi = \mu e^{(z-2)\ell}\nabla^2\phi - a e^{z\ell}\phi - b e^{(2\chi+z)\ell}\phi^3 + e^{(z-2\chi-d)\ell/2} f + \mu' e^{(z-4)\ell}\nabla^4\phi + b' e^{(4\chi+z)\ell}\phi^5 + \cdots$$

- Many terms are irrelevant upon coarse-graining

$$\partial_t\phi = \mu e^{(z-2)\ell}\nabla^2\phi - b e^{(2\chi+z)\ell}\phi^3 + e^{(z-2\chi-d)\ell/2} f$$

$$2\chi_{\rm lin} + z_{\rm lin} = 4 - d$$

$$\frac{1}{\mu_\ell}\frac{d\mu_\ell}{d\ell} = [z - 2 + C_\mu(\mu, D, b)] \;,\; \frac{1}{D_\ell}\frac{dD_\ell}{d\ell} = [z - 2\chi - d + C_D(\mu, D, b)] \;\; \frac{1}{b_\ell}\frac{db_\ell}{d\ell} = [2\chi + z + C_b(\mu, D, b)]$$

- Dimensional analysis: $[\phi] = [U] \quad , \quad [t] = [T] \quad , \quad [x] = [L]$

$$\Rightarrow \quad [\mu] = [L]^{-2} \quad , \quad [b] = [U]^{-2}[T] \quad , \quad [D] = [U]^2[T]^{-1}[L]^d$$

Dimensionless coefficient: $$g(\ell) \equiv \frac{b_\ell D_\ell}{\mu_\ell^2}\Lambda^{d-4}$$

Critical (scale invariant) dynamics 2

- How about the nonlinearities?

$$\partial_t \phi = \mu e^{(z-2)\ell} \nabla^2 \phi - a e^{z\ell} \phi - b e^{(2\chi+z)\ell} \phi^3 + e^{(z-2\chi-d)\ell/2} f + \mu' e^{(z-4)\ell} \nabla^4 \phi + b' e^{(4\chi+z)\ell} \phi^5 + \cdots$$

- Many terms are irrelevant upon coarse-graining

$$\partial_t \phi = \mu e^{(z-2)\ell} \nabla^2 \phi - b e^{(2\chi+z)\ell} \phi^3 + e^{(z-2\chi-d)\ell/2} f$$

$$2\chi_{\rm lin} + z_{\rm lin} = 4 - d$$

$$\frac{1}{\mu_\ell} \frac{d\mu_\ell}{d\ell} = [z - 2 + C_\mu(g_\ell)] \quad \frac{1}{D_\ell} \frac{dD_\ell}{d\ell} = [z - 2\chi - d + C_D(g_\ell)] \quad \frac{1}{b_\ell} \frac{db_\ell}{d\ell} = [2\chi + z + C_b(g_\ell)]$$

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Dimensionless coefficient: $$g(\ell) \equiv \frac{b_\ell D_\ell}{\mu_\ell^2} \Lambda^{d-4}$$

Ising universality class

$$g(\ell) \equiv \frac{b_\ell D_\ell}{\mu_\ell^2} \Lambda^{-\epsilon}$$

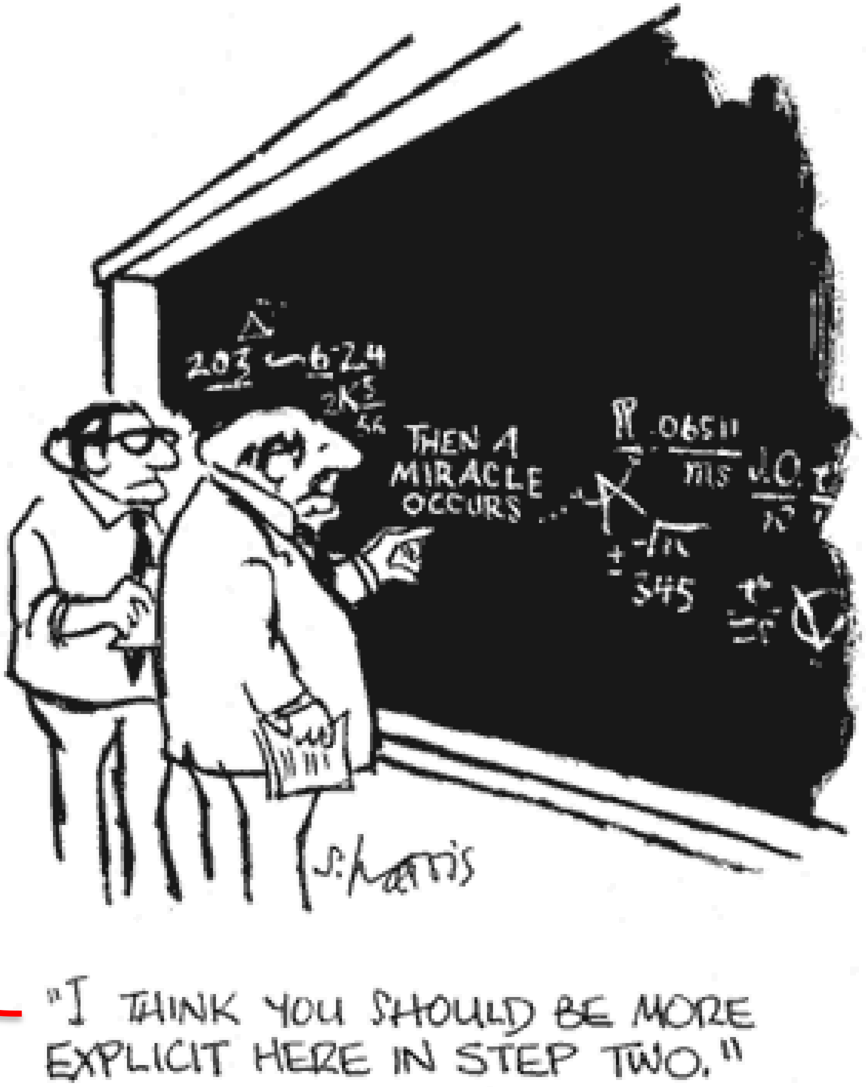
$$\epsilon = 4 - d$$

$$\frac{\mathrm{d}g}{\mathrm{d}\ell} = \;?$$

DRG

$\lim_{l \to \infty} g = A\epsilon$

Dynamical Renormalization Group



- N. Goldenfeld (1992) *Lectures On Phase Transitions And The Renormalization Group*
- J. Cardy (1996) *Scaling and Renormalization in Statistical Physics*
- M. Kardar (2007) *Statistical Physics of Fields*

Ising universality class

$$g(\ell) \equiv \frac{b_\ell D_\ell}{\mu_\ell^2}\Lambda^{-\epsilon}$$

$$\epsilon = 4 - d$$

$$\frac{\mathrm{d}g}{\mathrm{d}\ell} = \,?$$

DRG →

$$\lim_{l\to\infty} g = A\epsilon$$

$$\partial_t\phi = \mu \mathrm{e}^{z-2}\nabla^2\phi - b\mathrm{e}^{2\chi+z}\phi^3 + \mathrm{e}^{(z-2\chi-d)/2} f$$

Three equations, one constraint & two variables

$$\Rightarrow \quad z = 2 \quad , \quad \chi = -\frac{2-\epsilon}{2}$$

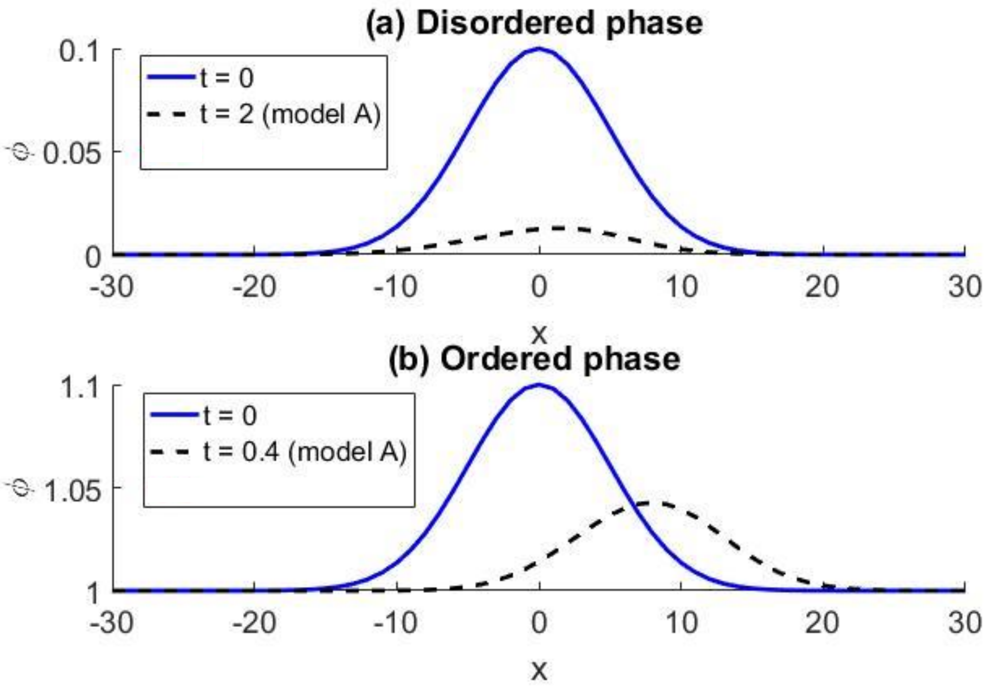
K.G. Wilson & M.E. Fisher (1972)
Critical Exponents in 3.99 Dimensions. Phys. Rev. Lett.

SYMMETRY-BASED ACTIVE ISING MODEL

Active Ising model

- Activity: Field variable coupled to spatial coordinate
  -> Coupled $\boldsymbol{Z}_2$-symmetry: $\phi \mapsto -\phi \quad , \quad x \mapsto -x$

- Generic EOM: $\partial_t \phi + \lambda \phi \partial_x \phi = (\mu_x \partial_x^2 + \mu_\perp \nabla_\perp^2)\phi - a\phi - b\phi^3 + f$



Universality in this active model

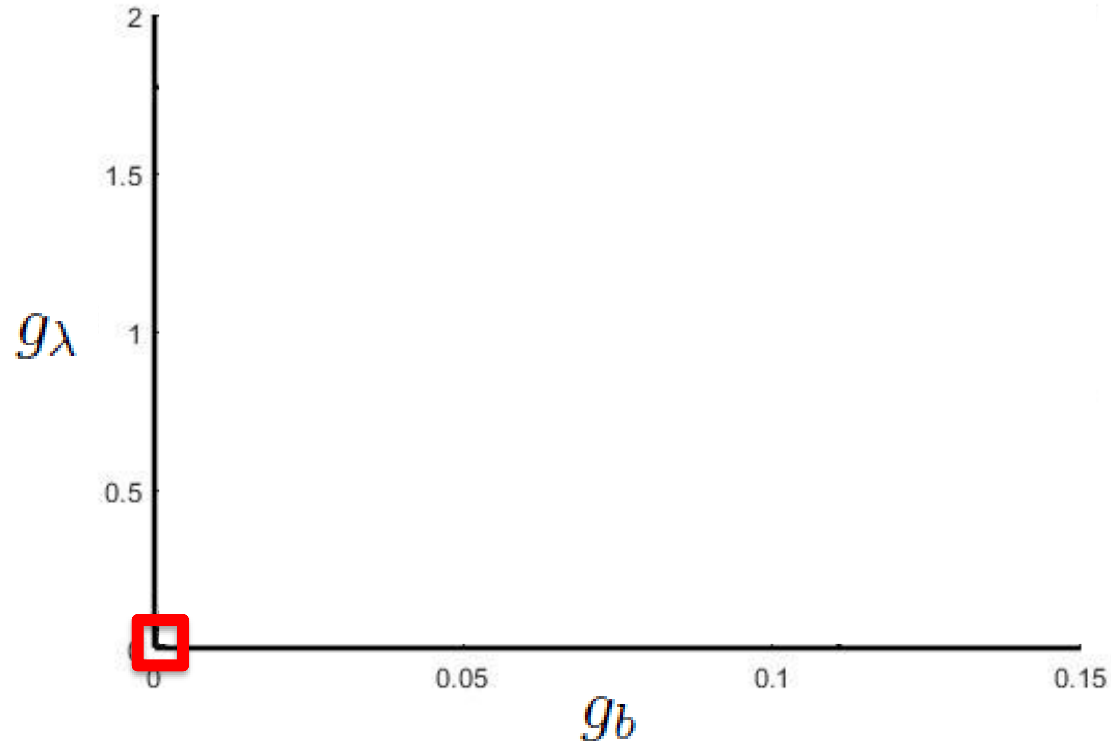
- Generic EOM: $\partial_t \phi + \lambda \phi \partial_x \phi = (\mu_x \partial_x^2 + \mu_\perp \nabla_\perp^2)\phi - a\phi - b\phi^3 + f$
- Anisotropic scaling:

$$\mathbf{x}_\perp \to \mathbf{x}_\perp e^{\ell} \quad , \quad x \to x e^{\zeta \ell} \quad , \quad t \to t e^{z\ell} \quad , \quad \phi \to e^{\chi \ell} \phi \; .$$

- At criticality, TWO coefficients become relevant at $d = 4$
- Two dimensionless coefficients:

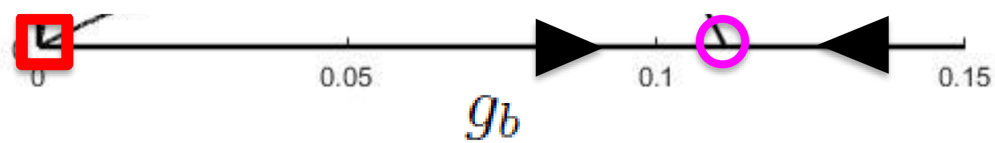
$$g_b \equiv \frac{bD}{\mu_x^{1/2} \mu_\perp^{3/2}} \Lambda^{d-4}$$

$$g_\lambda \equiv \frac{D\lambda^2}{\mu_x^{3/2} \mu_\perp^{3/2}} \Lambda^{d-4}$$



"Interface dynamics with up-down symmetry"
S.F. Edwards & D.R. Wilkinson (1982)
Proc. Royal Soc. London A

K.E. Bassler & B. Schmittmann (1994) Phys. Rev. Lett.



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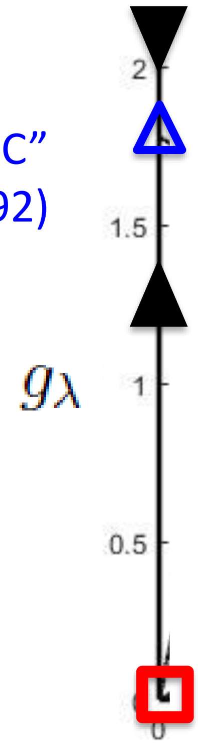
T. Hwa & M. Kardar, PRA (1992)



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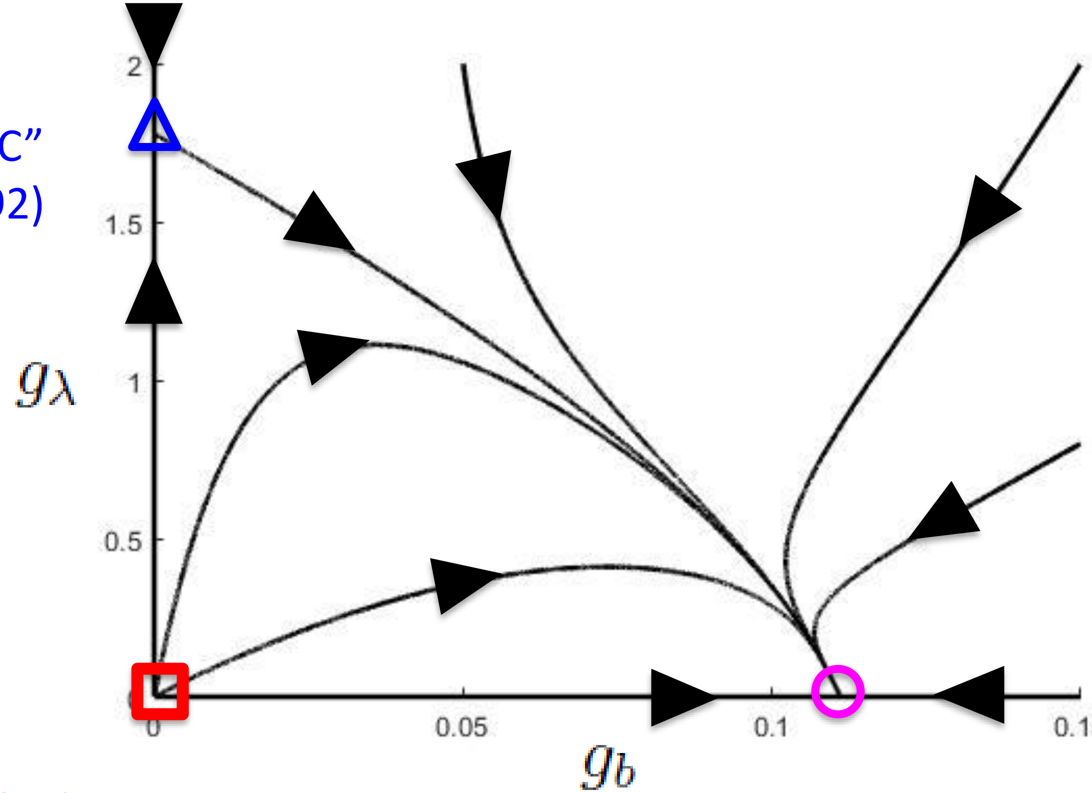

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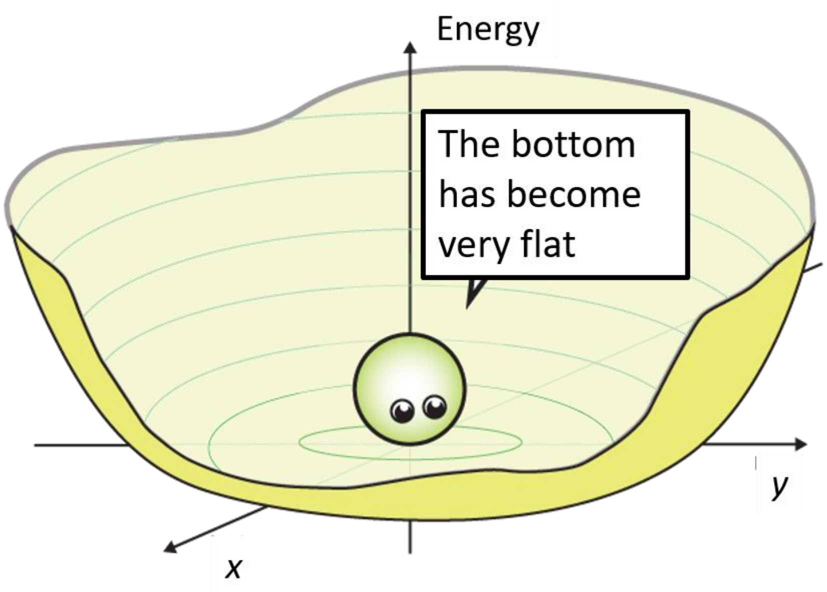
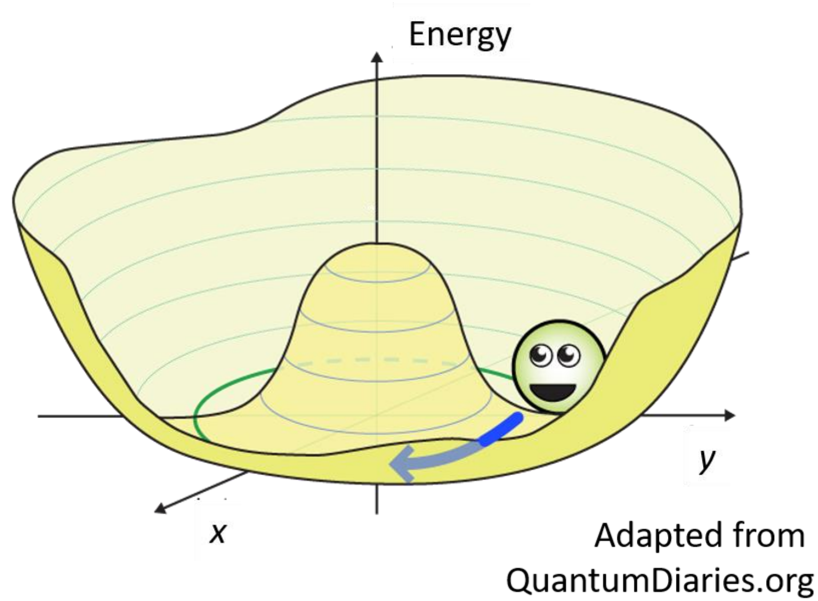
K.E. Bassler & B. Schmittmann (1994) Phys. Rev. Lett.

Z_2-symmetry: summary

- Equilibrium Ising: invariant under $\phi \mapsto -\phi$
- Active Ising: Invariant under $\phi \mapsto -\phi \quad , \quad x \mapsto -x$
- Active Ising (model A) = equilibrium Ising UC (1972)





2. Continuous symmetry: SO(d)-symmetry

Incompressible Navier-Stokes equation

- One coarse-grained field: $\mathbf{v}$
- EOM: $\partial_t \vec{v} = \mathcal{F}$
- Temporal invariance: $\mathcal{F}$ does not depend on t
- Translational inv.: $\mathcal{F}$ does not depend on $\mathbf{r}$
- Rotational inv.: Only diff. operators like ∇^2, ∇^4, etc, are in $\mathcal{F}$
- Parity inv.: EOM invariant under spatial inversion

$$\partial_t \vec{v} = -\vec{\nabla} P + \vec{f} - \lambda\left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} - (a + bv^2)\vec{v} - \mu \nabla^2 \vec{v} + cv^4 \vec{v} + \xi (\nabla^2)^2 \vec{v} + \cdots$$

- Galilean invariance: $\partial_t \vec{v} = -\vec{\nabla} P + \vec{f} - \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} - \mu \nabla^2 \vec{v}$

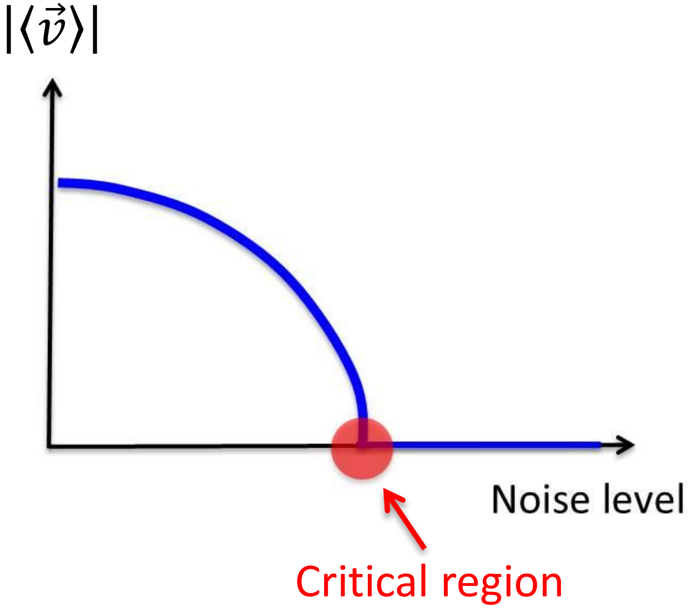
SYMMETRY-BASED ACTIVE FLUIDS

Lack of Galilean symmetry = incompressible active fluids

- All the previous symmetries except Galilean symmetry

$$\partial_t \vec{v} = -\vec{\nabla} P + \vec{f} - \lambda(\vec{v} \cdot \vec{\nabla})\vec{v} - (a + bv^2)\vec{v} - \mu\nabla^2\vec{v} + cv^4\vec{v} + \xi(\nabla^2)^2\vec{v} + \cdots$$

- Identical to the Toner-Tu flocking model in the incompressible limit [Toner & Tu (1998) Phys. Rev. E]



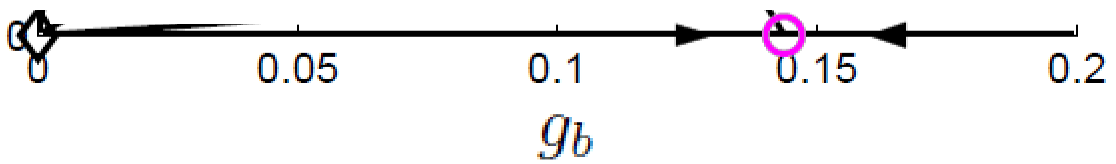
Critical dynamics

- Linear EOM: $\partial_t \mathbf{v} = \mathbf{f} - \nabla P + \mu \nabla^2 \mathbf{v}$

- Nonlinearities: both λ and b become relevant at $d = 4$

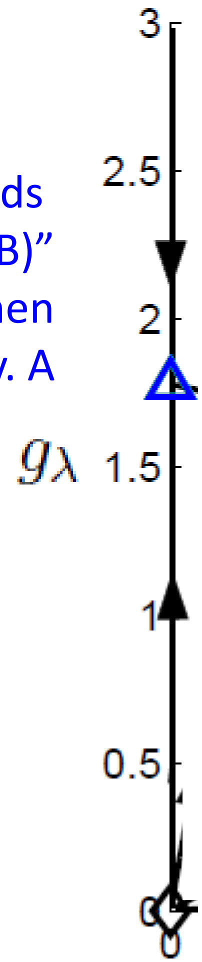
$$g_\lambda = \frac{\lambda^2 D}{\mu^3} \Lambda^{d-4}$$

$$g_b = \frac{bD}{\mu^2} \Lambda^{d-4}$$



“Ferromagnets with dipolar interactions”
Aharony and Fisher (1973) Phys. Rev. Lett.

L. Chen, J. Toner, CFL (2015)
New J. Phys. **17**, 042002



"Randomly stirred fluids (Model B)"
Forster, Nelson & Stephen (1977) Phys. Rev. A

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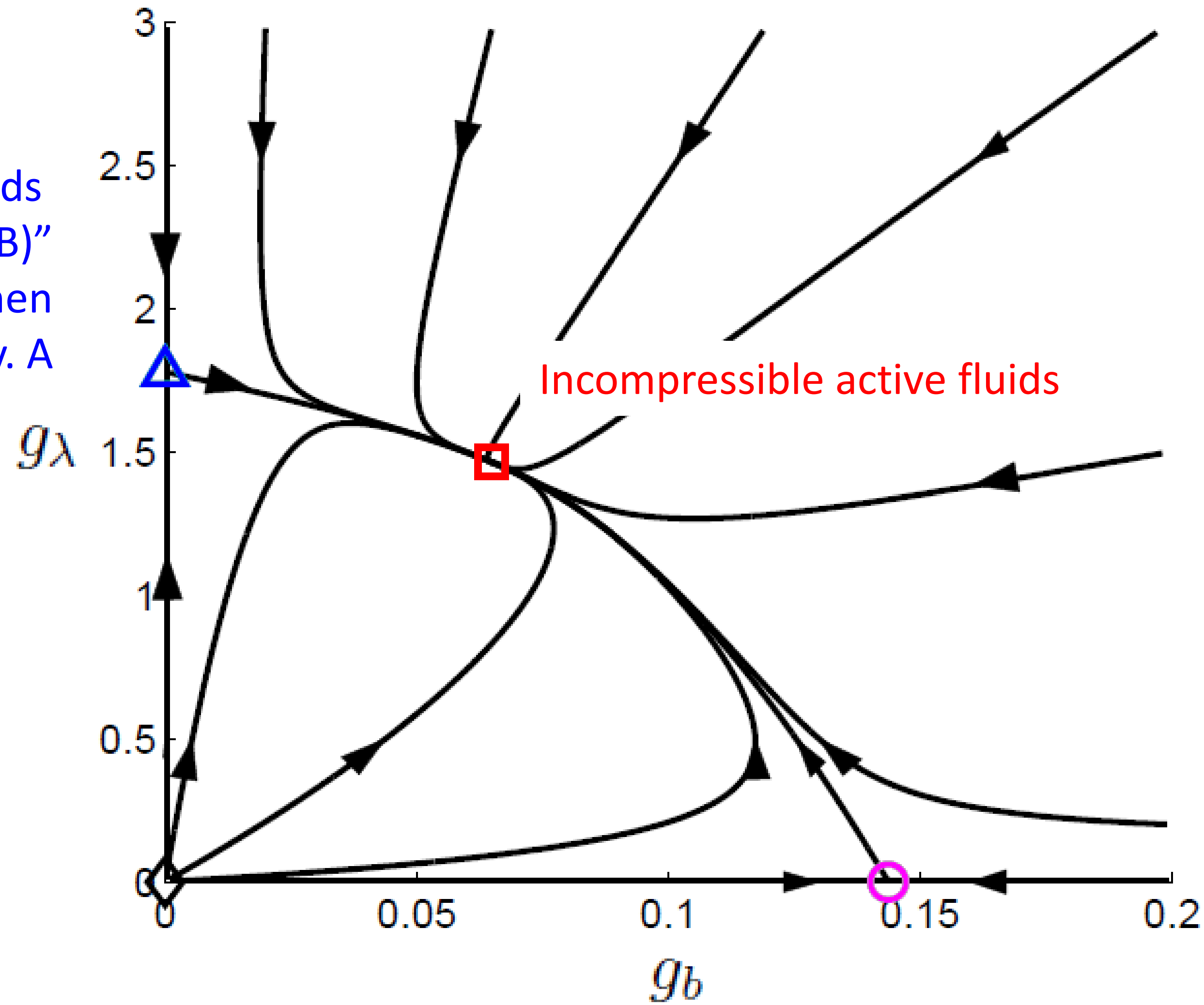

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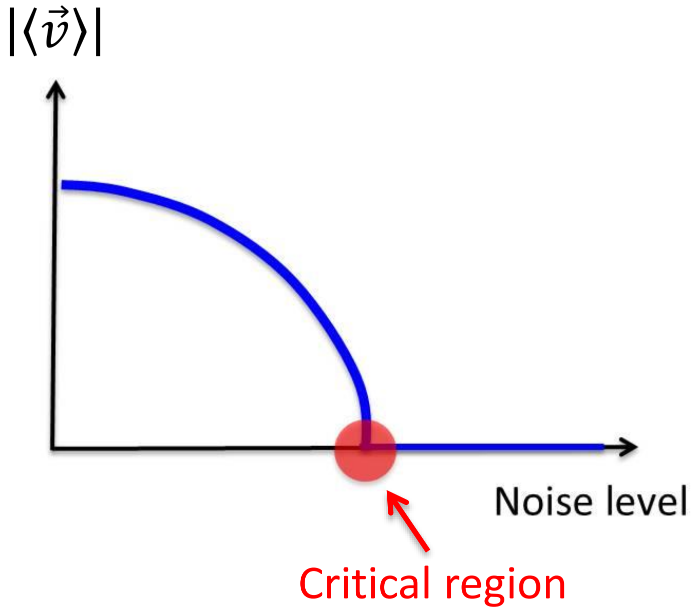
Ordered phase



Ordered phase in 2D

- Collective motion along the x direction: $\mathbf{v} = (v_0 + u_x)\hat{\mathbf{x}} + \mathbf{u}_\perp$

- Linear EOM:

$$\partial_t \mathbf{u} = \mathbf{f} - \nabla P + \mu_x \nabla^2 u_x + \mu_\perp \nabla_\perp^2 \mathbf{u}_\perp - \lambda v_0 \partial_x \mathbf{u} + 2 a v_0 u_x \hat{\mathbf{x}}$$

- λ is irrelevant -> the model can be mapped onto an equilibrium system

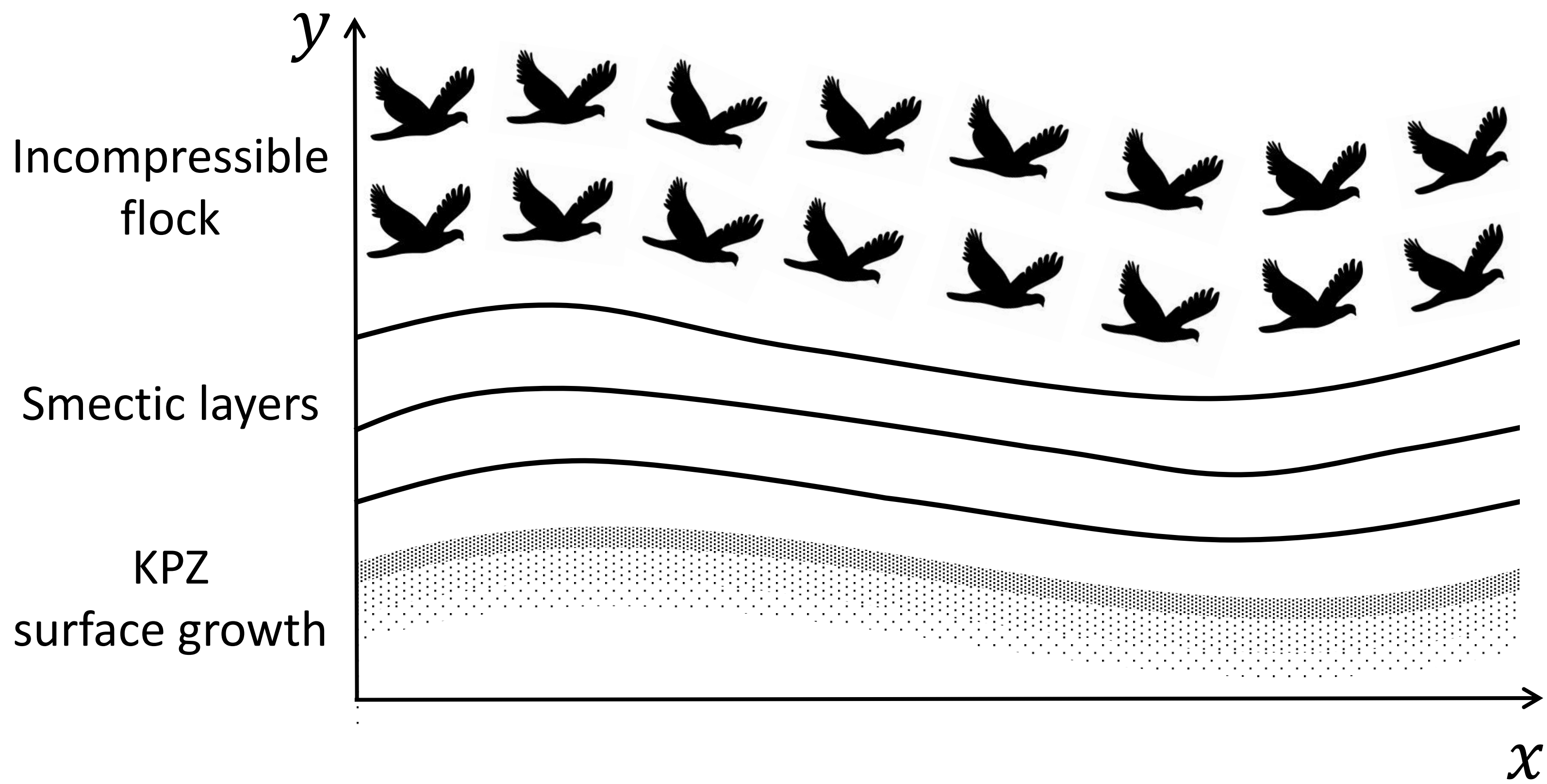
- Mermin-Wagner theorem in 2D?

- $\nabla \cdot \vec{v} = 0$ -> $k_x v_x = -k_y v_y$

- Just enough constraint to bypass MW Theorem & give nontrivial universal scaling

L. Chen, CFL, J. Toner (2016) Nature Comm. 7, 12215

KPZ universality class



L. Chen, CFL, J. Toner (2016) Nature Comm. 7, 12215

Ordered phase in 3D

- Collective motion along the x direction: $\mathbf{v} = (v_0 + u_x)\hat{\mathbf{x}} + \mathbf{u}_\perp$
- Linear EOM:

$$\partial_t \mathbf{u} = \mathbf{f} - \nabla P + \mu_x \nabla^2 u_x + \mu_\perp \nabla_\perp^2 \mathbf{u}_\perp - \lambda v_0 \partial_x \mathbf{u} + 2 a v_0 u_x \hat{\mathbf{x}}$$

- λ is relevant but a is not -> the model remains non-equilibrium
- One dimensionless coefficient: $g_\lambda \equiv \frac{D\lambda^2}{\mu_x^{3/2}\mu_\perp^{3/2}} \Lambda^{d-4}$

Flocking universality class [Toner & Tu (1995) PRL]

L. Chen, CFL, J. Toner,
In preparation

$SO(d)$-symmetry: summary

- Incomp. active fluids (IAF) = Incomp. passive fluids without Galilean invariance and fluctuation-dissipation relation

- Critical IAF exhibits novel universal behaviour

- IAF in the ordered phase:
  - 2D: KPZ universality class
  - 3D: Flocking universality class

Main messages:

- Symmetry -> active matter
- Connections to diverse known physical systems
- Emergence of new universality classes

3. Summary & Outlook

Main messages:

- Symmetry -> active matter
- Connections to diverse known physical systems
- Emergence of new universality classes

Z_2-symmetry: summary

- Equilibrium Ising: invariant under $\phi \mapsto -\phi$
- Active Ising: Invariant under $\phi \mapsto -\phi \quad , \quad x \mapsto -x$

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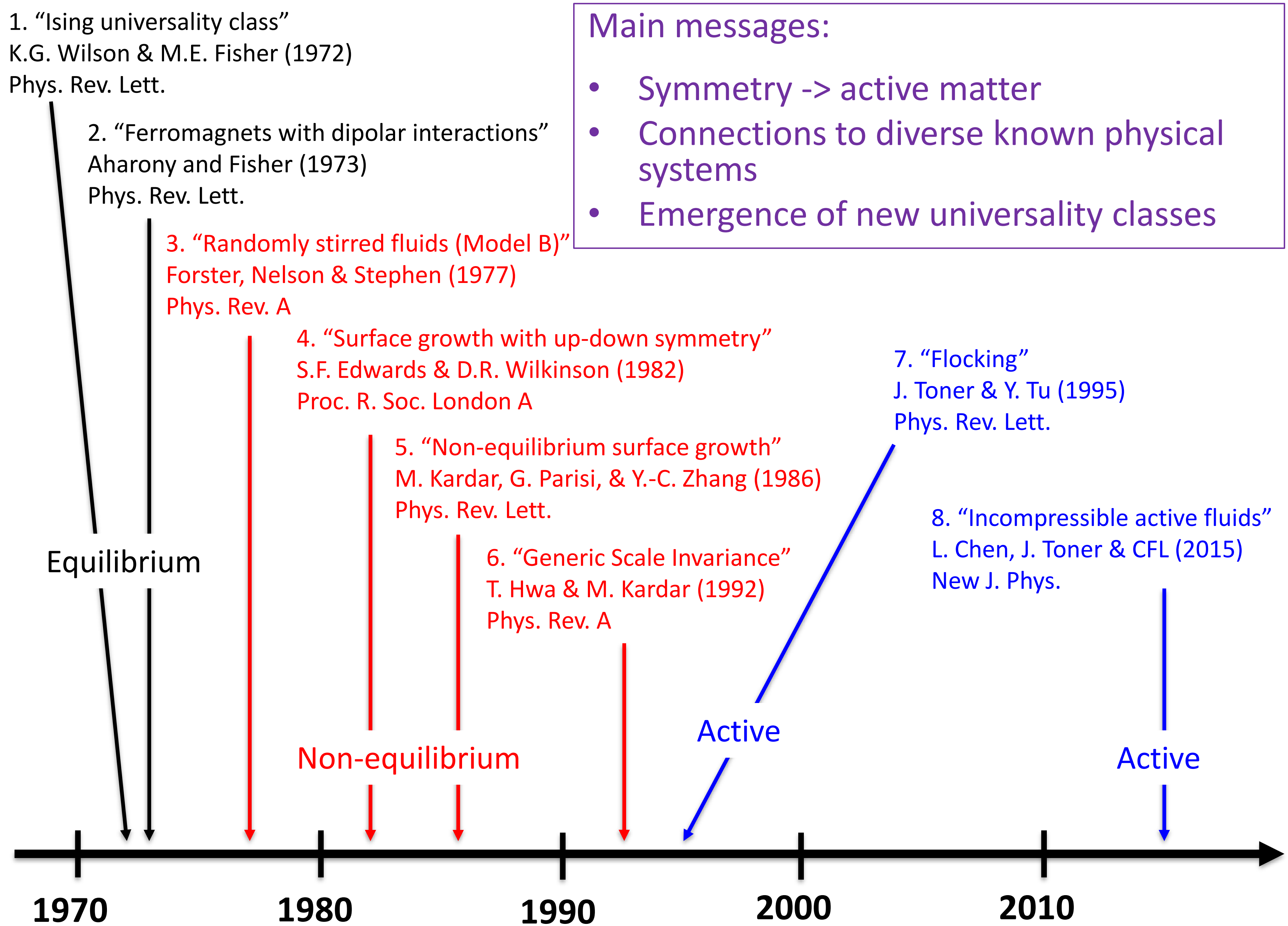
- All the previous symmetries except Galilean symmetry

$$\partial_t \vec{v} = -\vec{\nabla} P + \vec{f} - \lambda\left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} - (a + bv^2)\vec{v} - \mu \nabla^2 \vec{v} + cv^4 \vec{v} + \xi(\nabla^2)^2 \vec{v} + \cdots$$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1. "Ising universality class"
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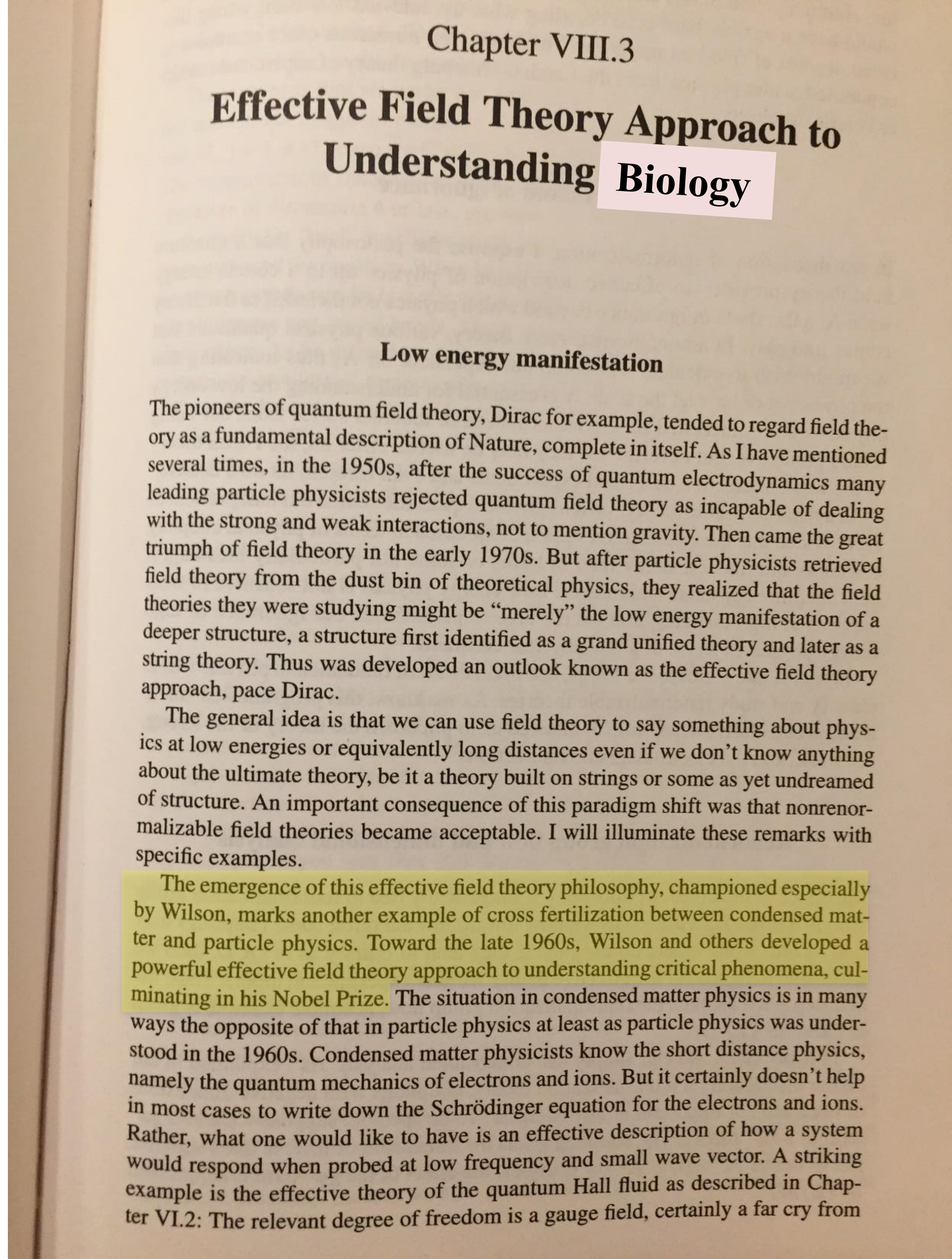
Phys. Rev. Lett.
2. "Ferromagnets with dipolar interactions"
Aharony and Fisher (1973)
Phys. Rev. Lett.
3. "Randomly stirred fluids (Model B)"
Forster, Nelson & Stephen (1977)
Phys. Rev. A
4. "Surface growth with up-down symmetry"
S.F. Edwards & D.R. Wilkinson (1982)
Proc. R. Soc. London A
5. "Non-equilibrium surface growth"
M. Kardar, G. Parisi, & Y.-C. Zhang (1986)
Phys. Rev. Lett.
6. "Generic Scale Invariance"
T. Hwa & M. Kardar (1992)
Phys. Rev. A
7. "Flocking"
J. Toner & Y. Tu (1995)
Phys. Rev. Lett.
8. "Incompressible active fluids"
L. Chen, J. Toner & CFL (2015)
New J. Phys.
Equilibrium
Non-equilibrium
Active
Active
1970
1980
1990
2000
2010

Outlook

Anthony Zee, Quantum Field Theory in a Nutshell (Princeton University Press, 2003)



Chapter VIII.3

Effective Field Theory Approach to Understanding Biology

Low energy manifestation

The pioneers of quantum field theory, Dirac for example, tended to regard field theory as a fundamental description of Nature, complete in itself. As I have mentioned several times, in the 1950s, after the success of quantum electrodynamics many leading particle physicists rejected quantum field theory as incapable of dealing with the strong and weak interactions, not to mention gravity. Then came the great triumph of field theory in the early 1970s. But after particle physicists retrieved field theory from the dust bin of theoretical physics, they realized that the field theories they were studying might be "merely" the low energy manifestation of a deeper structure, a structure first identified as a grand unified theory and later as a string theory. Thus was developed an outlook known as the effective field theory approach, pace Dirac.

The general idea is that we can use field theory to say something about physics at low energies or equivalently long distances even if we don't know anything about the ultimate theory, be it a theory built on strings or some as yet undreamed of structure. An important consequence of this paradigm shift was that nonrenormalizable field theories became acceptable. I will illuminate these remarks with specific examples.

The emergence of this effective field theory philosophy, championed especially by Wilson, marks another example of cross fertilization between condensed matter and particle physics. Toward the late 1960s, Wilson and others developed a powerful effective field theory approach to understanding critical phenomena, culminating in his Nobel Prize. The situation in condensed matter physics is in many ways the opposite of that in particle physics at least as particle physics was understood in the 1960s. Condensed matter physicists know the short distance physics, namely the quantum mechanics of electrons and ions. But it certainly doesn't help in most cases to write down the Schrödinger equation for the electrons and ions. Rather, what one would like to have is an effective description of how a system would respond when probed at low frequency and small wave vector. A striking example is the effective theory of the quantum Hall fluid as described in Chapter VI.2: The relevant degree of freedom is a gauge field, certainly a far cry from

Two upcoming events at Imperial College

‘Field theories come to Life’ Workshop (9 April 2018)

Speakers:

Mike Cates (Cambridge)
Erwin Frey (TU München)
Nathan Goehring (Crick Institute)
Guillaume Salbreux (Crick Institute)
Kay Jörg Wiese (ENS Paris)
Frédéric van Wijland (U Paris Diderot)

EPSRC Fluids CDT summer school ‘Current topics in active fluids: theory and experiments’ (16-20 July 2018)

Keynotes Lecturers:

Peer Fischer (Max Planck Institute for Intelligent Systems)
Ewa Paluch (UCL)
Mike Shelley (New York University)
John Toner (University of Oregon)
Julia Yeomans (Oxford)

Thank you for your attention