Universality in Living Matter

Chiu Fan Lee

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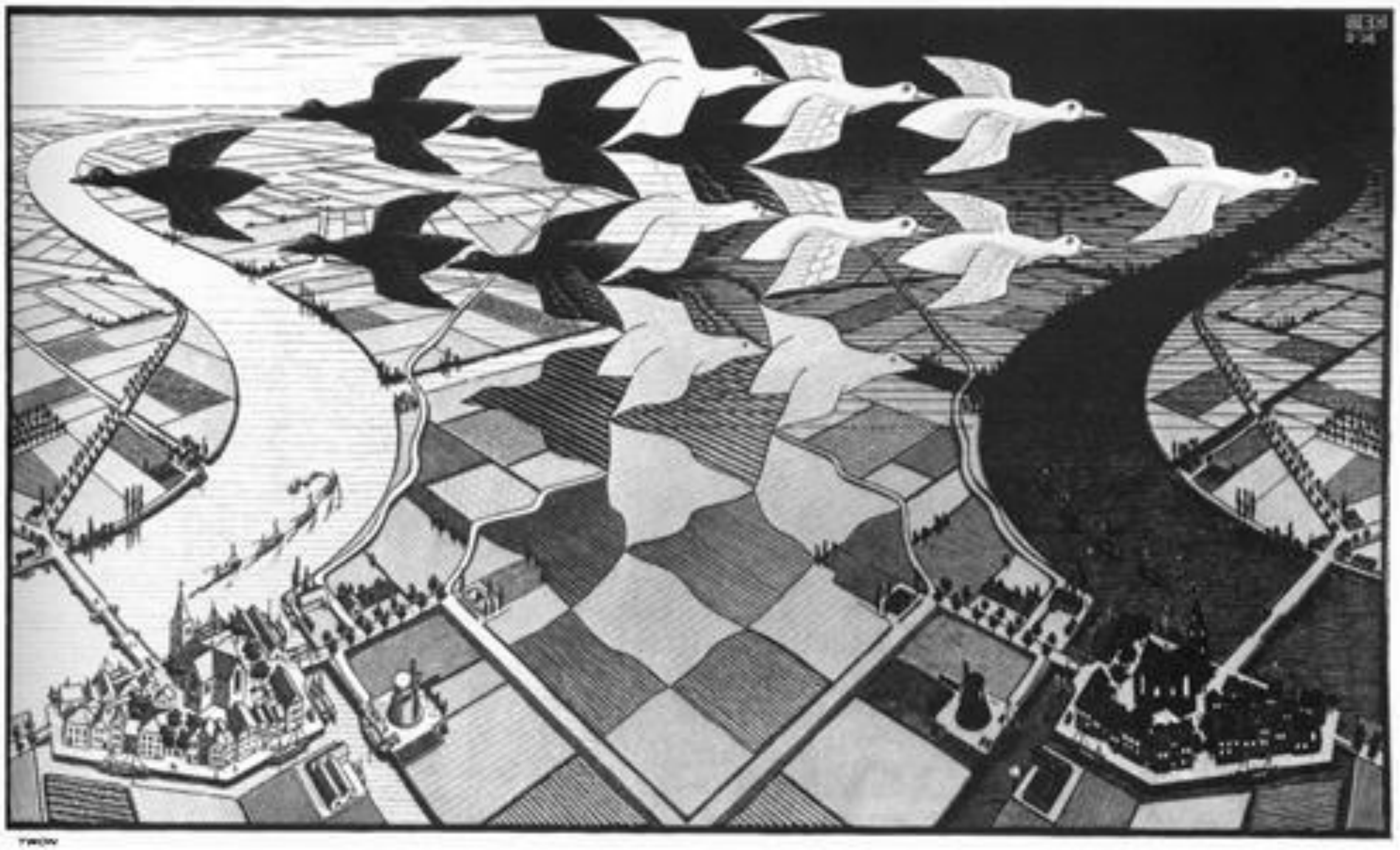
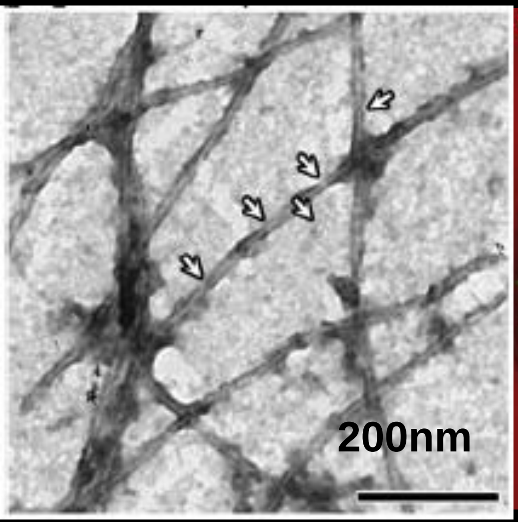


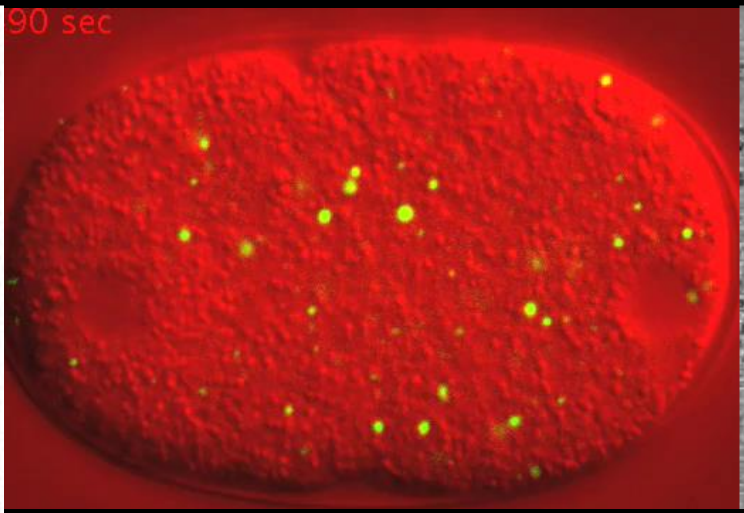
M.C. Escher (1938) *Day and Night*

Universalities in soft living matter

Amyloid fibrils

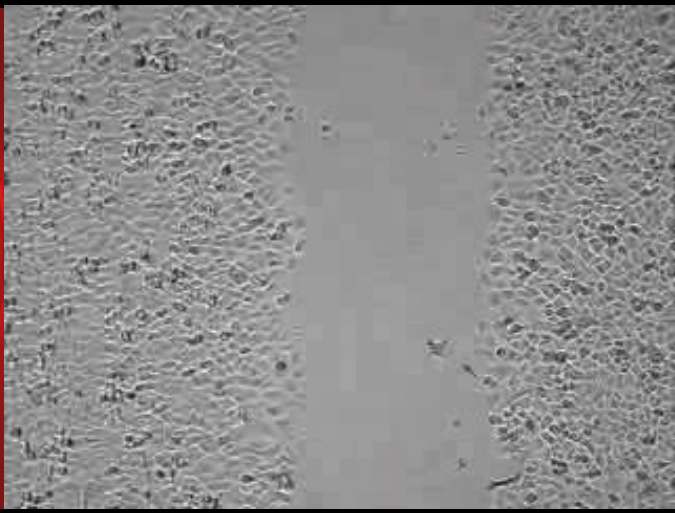


RNA granules



Brangwynne [Hyman Lab]

Active matter



https://www.youtube.com/watch?v=EimBzUSmak8

Equilibrium: length distribution, breakage profile

Equilibrium: Lifshitz-Slyozov scaling & drop size distribution

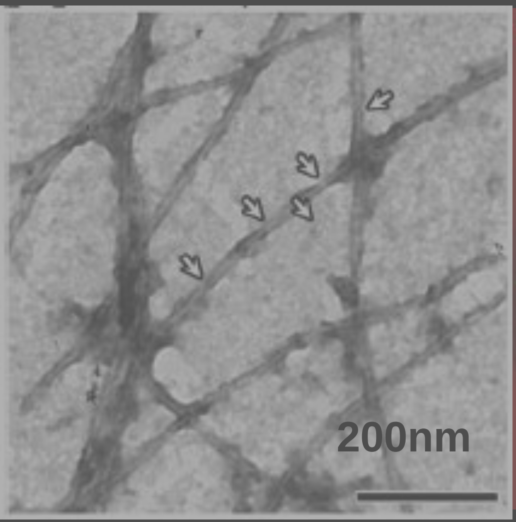
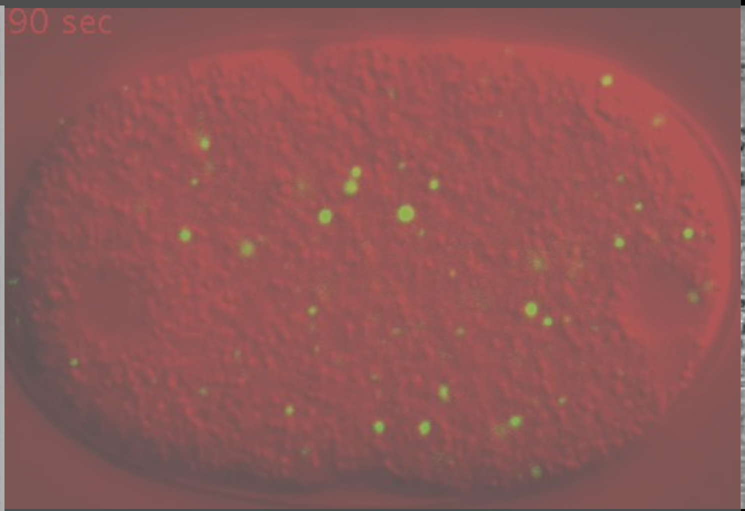
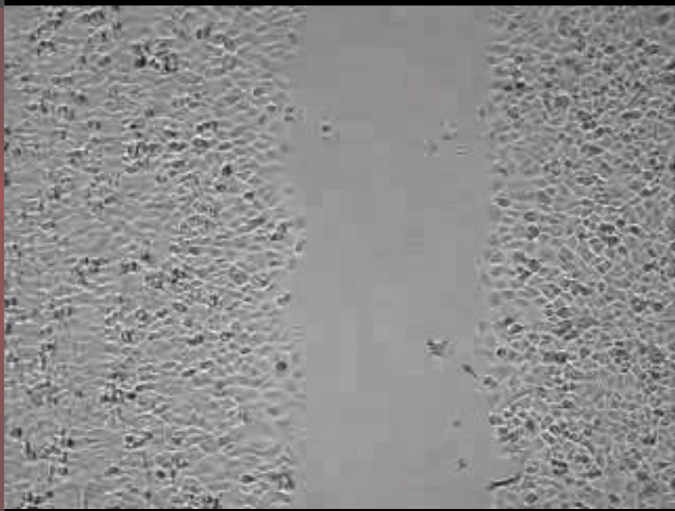
Fundamentally non-equilibrium

Non-equilibrium universality?

Non-equilibrium universality?

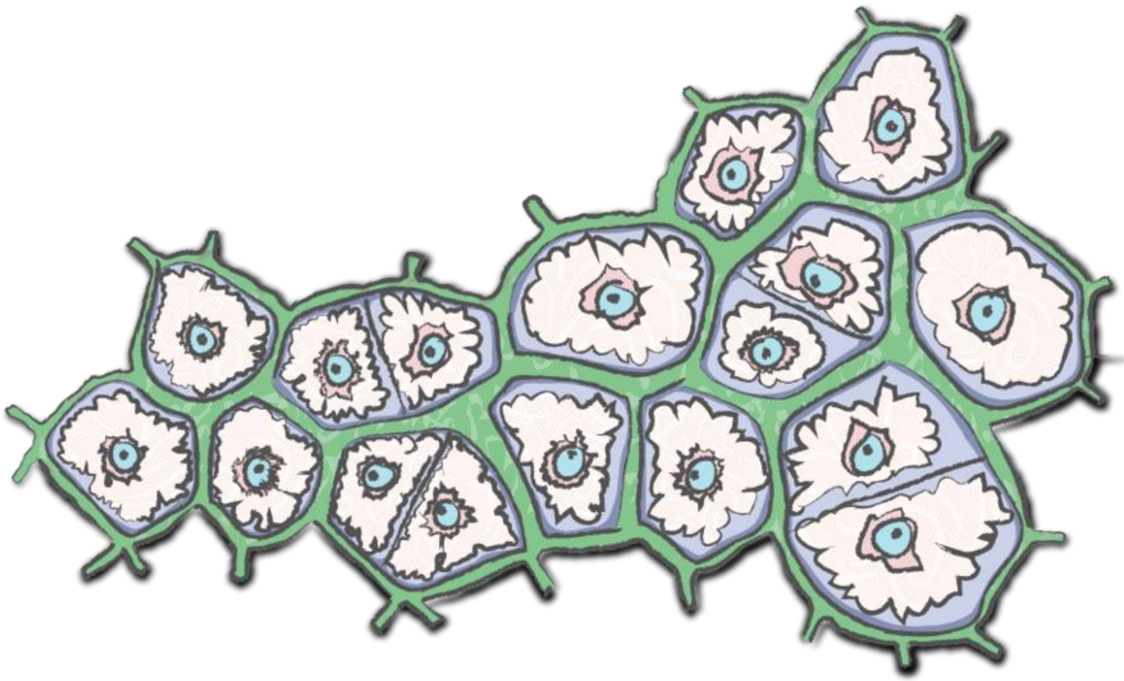
Universality?

Universalities in living matter

Amyloid fibrils	RNA granules	Active matter
		
	Brangwynne [Hyman Lab]	https://www.youtube.com/watch?v=EimBzUSmak8
Equilibrium: length distribution, breakage profile	Equilibrium: Lifshitz-Slyozov scaling & drop size distribution	Fundamentally non-equilibrium
Non-equilibrium universality?	Non-equilibrium universality?	Universality?

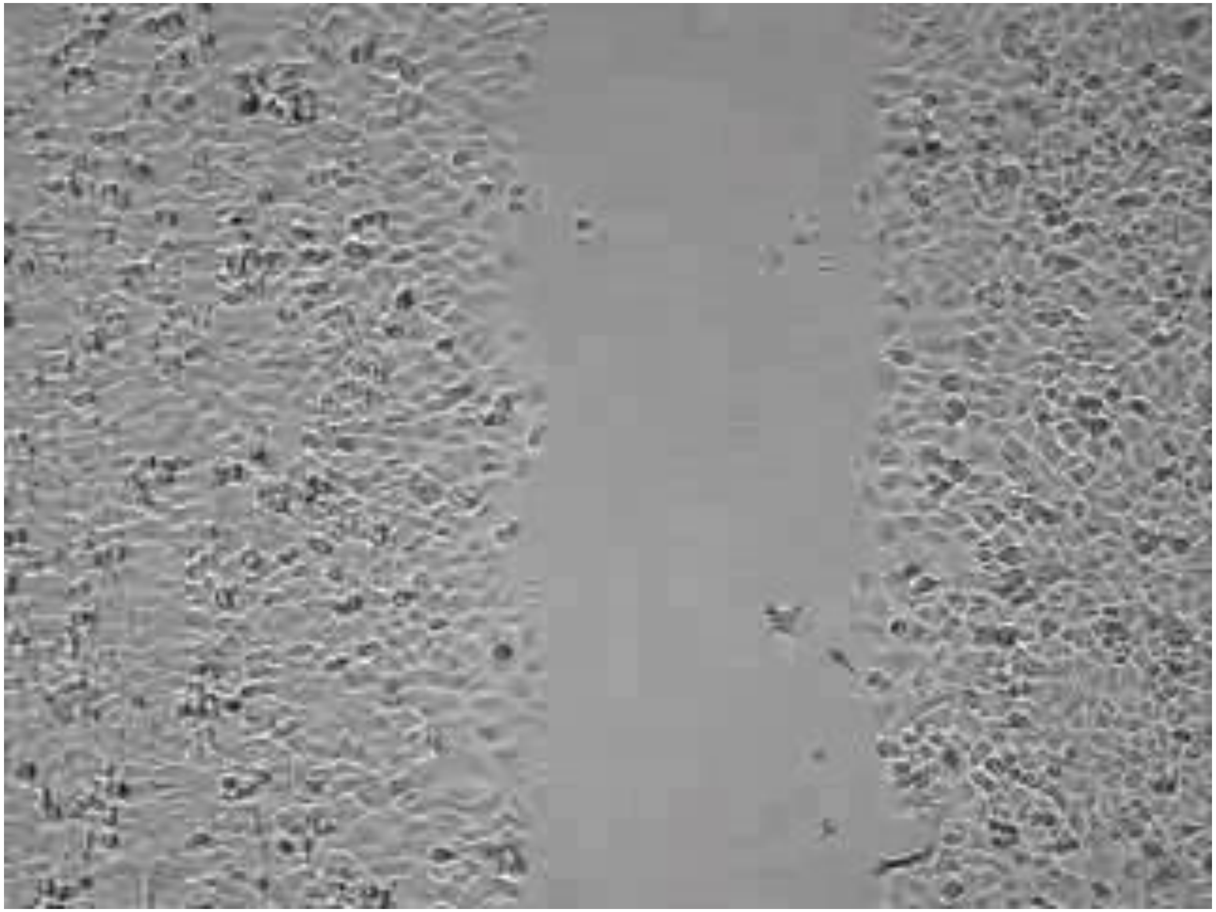
Plan

- What are incompressible active fluids?
- Concept of universality
- Universality of incompressible active fluids
  - Critical order-disorder transition
  - Ordered phase (2D)
- Active polymer networks
- Outlook & summary



What are incompressible active fluids?

Wound healing assay



https://www.youtube.com/watch?v=v9xq_GiRXeE

Navier-Stokes description?

$$\partial_t \vec{v} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} = \mu \nabla^2 \vec{v} - \rho^{-1} \vec{\nabla} P$$

Not enough!

Navier-Stokes description?

$$\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla})\vec{v} = (a + bv^2)\vec{v} + \mu \nabla^2 \vec{v} - \rho^{-1} \vec{\nabla} P$$

Not enough!

Missing elements:

- Cells are motile -> Need *active* terms like $(a + bv^2)\vec{v}$

Navier-Stokes description?

$$\partial_t \vec{v} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} = (a + bv^2)\vec{v} + \mu \nabla^2 \vec{v} - \rho^{-1}\vec{\nabla} P + \vec{f}$$

Not enough!

Missing elements:

- Cells are motile -> Need *active* terms like $(a + bv^2)\vec{v}$
- Fluctuations can be important -> Gaussian noise $\vec{f}$

Navier-Stokes description?

$$\partial_t \vec{v} + \lambda(\vec{v} \cdot \vec{\nabla})\vec{v} = (a + bv^2)\vec{v} + \mu \nabla^2 \vec{v} - \rho^{-1} \vec{\nabla} P + \vec{f}$$

Not enough!

Missing elements:

- Cells are motile -> Need *active* terms like $(a + bv^2)\vec{v}$
- Fluctuations can be important -> Gaussian noise $\vec{f}$
- No Galilean invariance -> prefactor of $(\vec{v} \cdot \vec{\nabla})\vec{v}$ may not be 1

In fact, to be completely general…

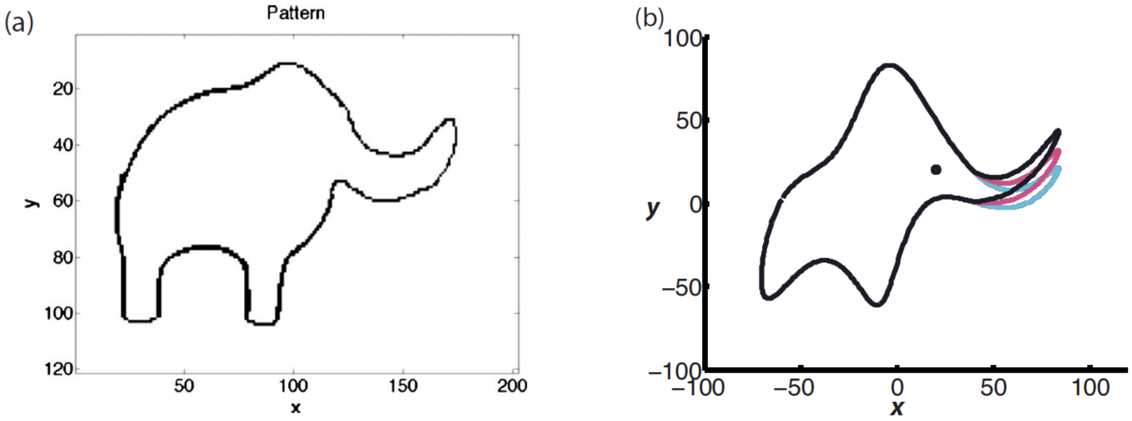
There can be infinitely many more red terms

$$\partial_t \vec{v} + \rho^{-1}\vec{\nabla}P - \vec{f} = -\lambda\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} - (a + bv^2)\vec{v} - \mu\nabla^2\vec{v}$$
$$+cv^4\vec{v} + \xi(\nabla^2)^2\vec{v} + \cdots$$

The perils of parameter fitting

1903-1957

- **John von Neumann**: *"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk."*



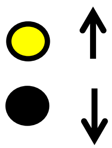
Mayer, Khairy & Howard (2010) Amer. J. Physics

With so many parameters, can we ever say something meaningful about the phenomenon?

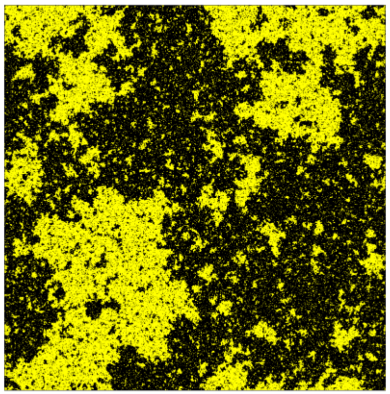
Concept of universality

Universality in physics

Ferromagnetism with an easy axis at criticality

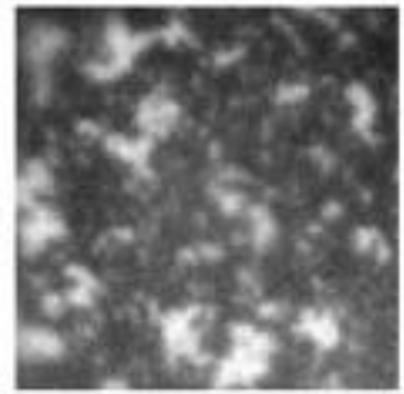


P. Colon's homepage



Phase separation in multi-component lipid membrane at criticality

Honerkamp-Smith et al. (2009) BBA



As the measurement is at a larger and larger length scale ($l \to \infty$)

Ising model: $E = -J \sum_{<i.j>} s_i \, s_j$

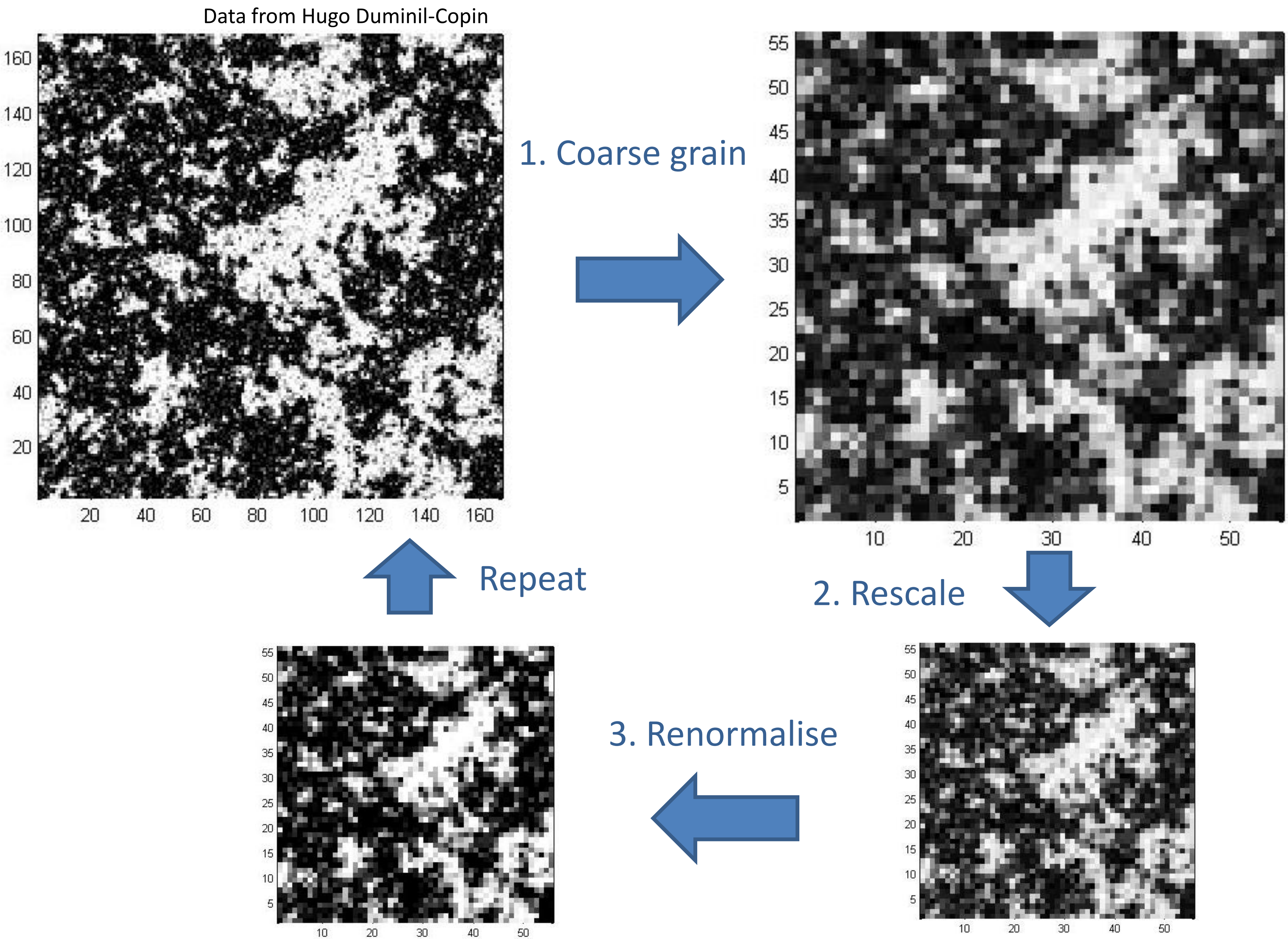
Proved by renormalisation group methods

Kenneth Wilson, Nobel prize 1982

Renormalisation group transformation

$$H_0 = \int d^d r \Big[(\nabla S(\mathbf{r}))^2 + t_0 S(\mathbf{r})^2 + u_0 S(\mathbf{r})^4 + g_6 S(\mathbf{r})^6 + \lambda_4 (\nabla S(\mathbf{r}))^4 + \dots \Big]$$

1. Coarse grain out short wavelength ($k > l^{-1}$) fluctuations
2. Rescale
3. Renormalise


Data from Hugo Duminil-Copin
1. Coarse grain
2. Rescale
3. Renormalise
Repeat

Renormalisation group transformation

$$H_0 = \int d^d r \left[(\nabla S(\mathbf{r}))^2 + t_0 S(\mathbf{r})^2 + u_0 S(\mathbf{r})^4 + g_6 S(\mathbf{r})^6 + \lambda_4 (\nabla S(\mathbf{r}))^4 + \dots \right]$$

1. Coarse grain out short wavelength ($k > l^{-1}$) fluctuations
2. Rescale
3. Renormalise

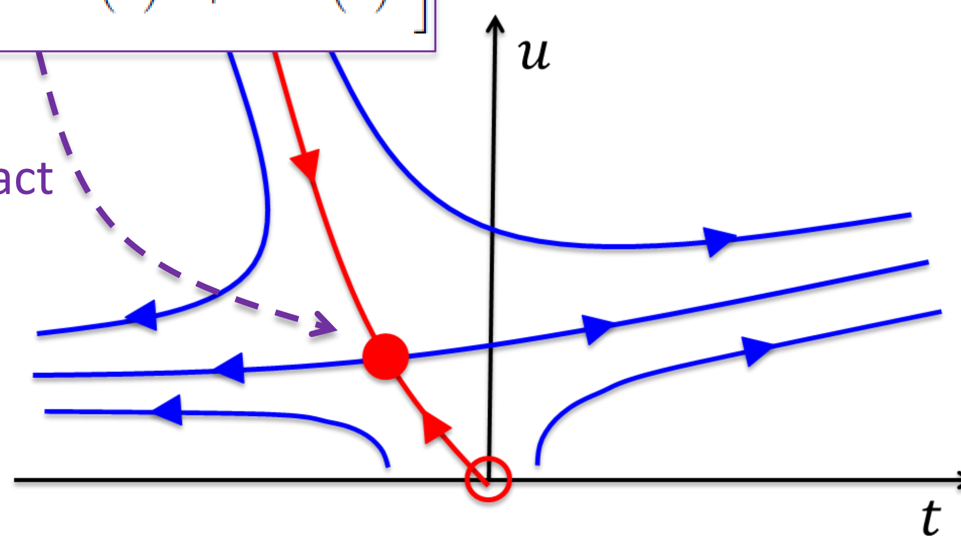
$$H_l = \int d^d r \left[(\nabla S(\mathbf{r}))^2 + t_l S(\mathbf{r})^2 + u_l S(\mathbf{r})^4 \right]$$

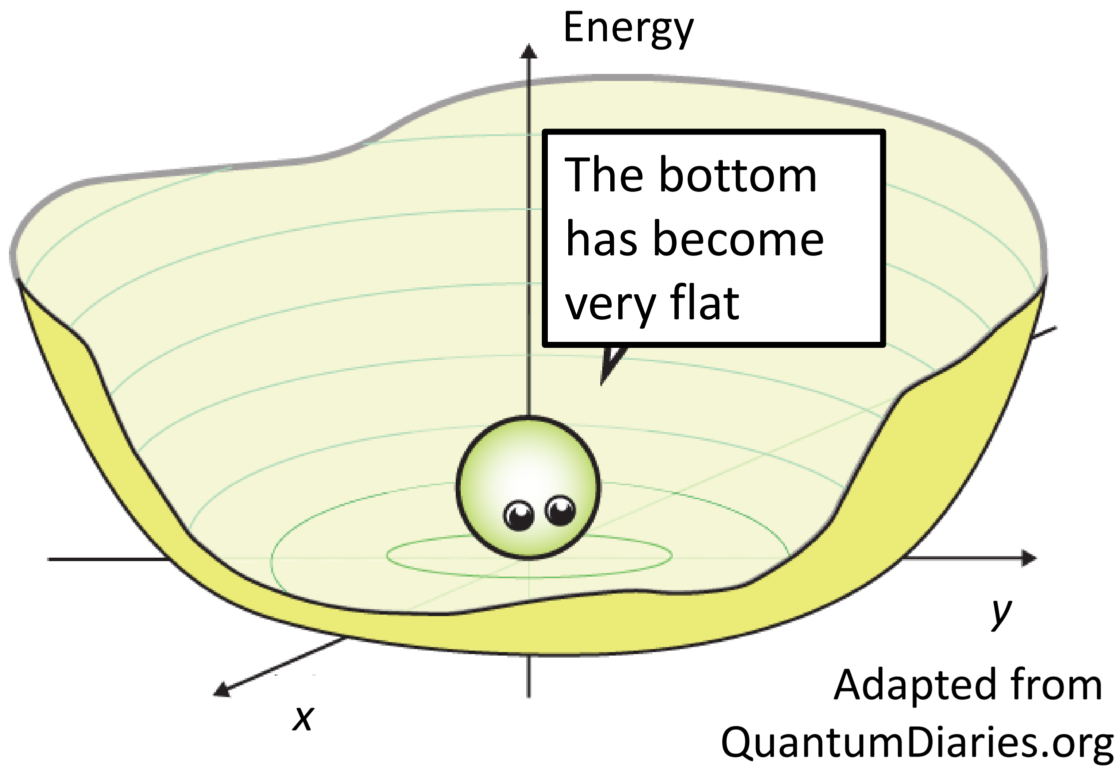
l goes up further

$$H^* = \int d^d r \left[(\nabla S(\mathbf{r}))^2 + t^* S(\mathbf{r})^2 + u^* S(\mathbf{r})^4 \right]$$

Toy model H^* becomes the (asymptotically) exact model

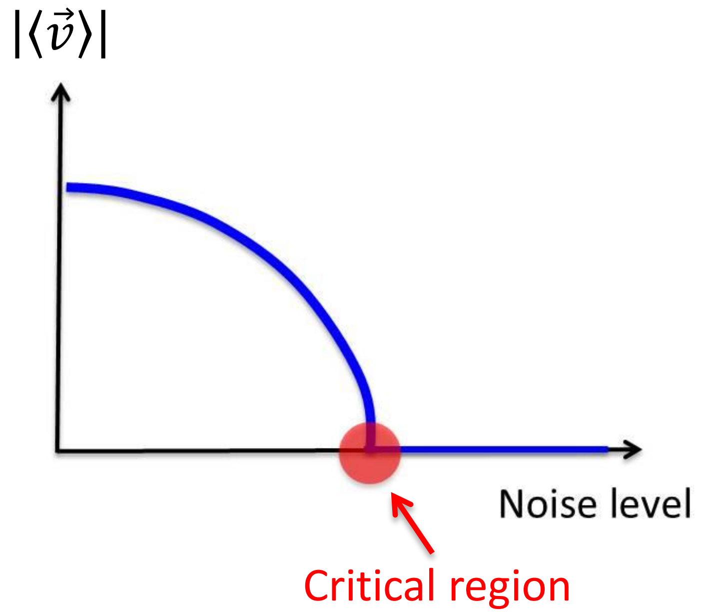
Level of coarse-graining





Critical order-disorder transition

Phase diagram of incompressible active fluids



Critical incompressible active fluids

EOM: $\partial_t \vec{v} + \vec{\nabla} P - \vec{f} = -\lambda(\vec{v} \cdot \vec{\nabla})\vec{v} - (a + bv^2)\vec{v} - \mu\nabla^2\vec{v} + cv^4\vec{v} + \xi(\nabla^2)^2\vec{v} + \cdots$

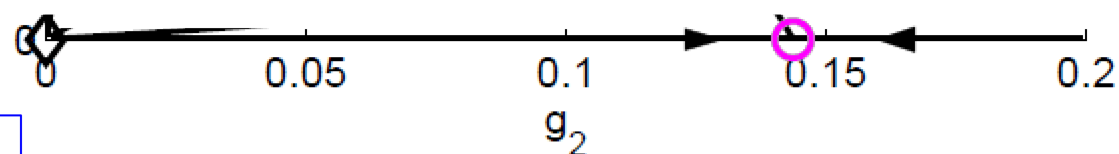
RG transformation

$$\partial_t \vec{v} + \vec{\nabla} P + \vec{f_l} = -\lambda_l(\vec{v} \cdot \vec{\nabla})\vec{v} - (a_l + b_l v^2)\vec{v} - \mu_l \nabla^2 \vec{v}$$

Exact hydrodynamic EOM with **TWO** coefficients governing the model's scale-invariance properties:

$$g_1(l) \sim \frac{D_l {\lambda_l}^2}{{\mu_l}^3} \quad , \quad g_2(l) \sim \frac{D_l b_l}{{\mu_l}^2}$$

Level of coarse-graining

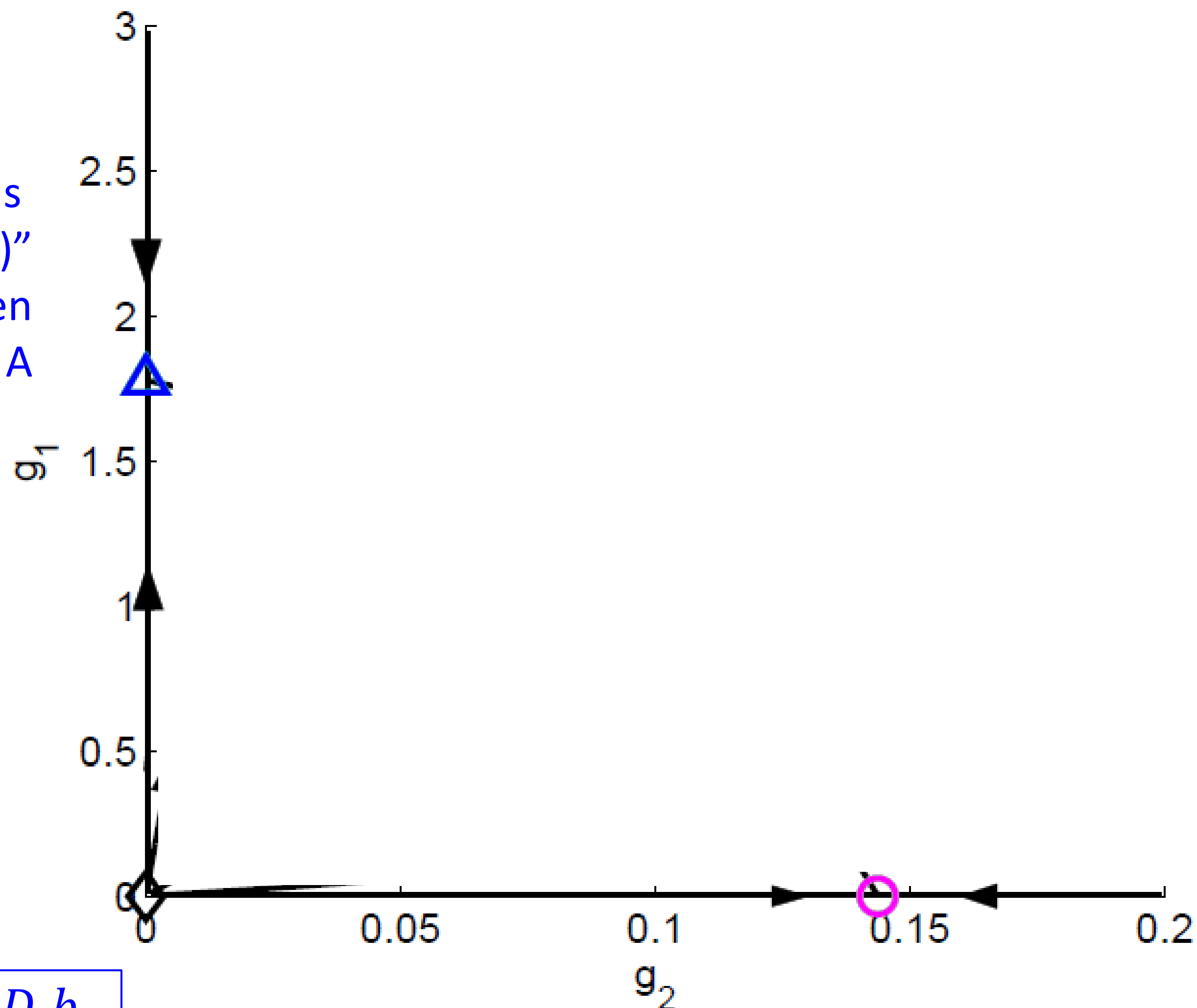


$$g_1(l) \sim \frac{D_l \lambda_l{}^2}{\mu_l{}^3};\; g_2(l) \sim \frac{D_l b_l}{\mu_l{}^2}$$

"Ferromagnets with dipolar interactions"
Aharony and Fisher (1973) Phys. Rev. Lett.

Chen, Toner, Lee (2015)
New J. Phys. **17**, 042002

“Randomly stirred fluids (Model B)”
Forster, Nelson & Stephen (1977) Phys. Rev. A



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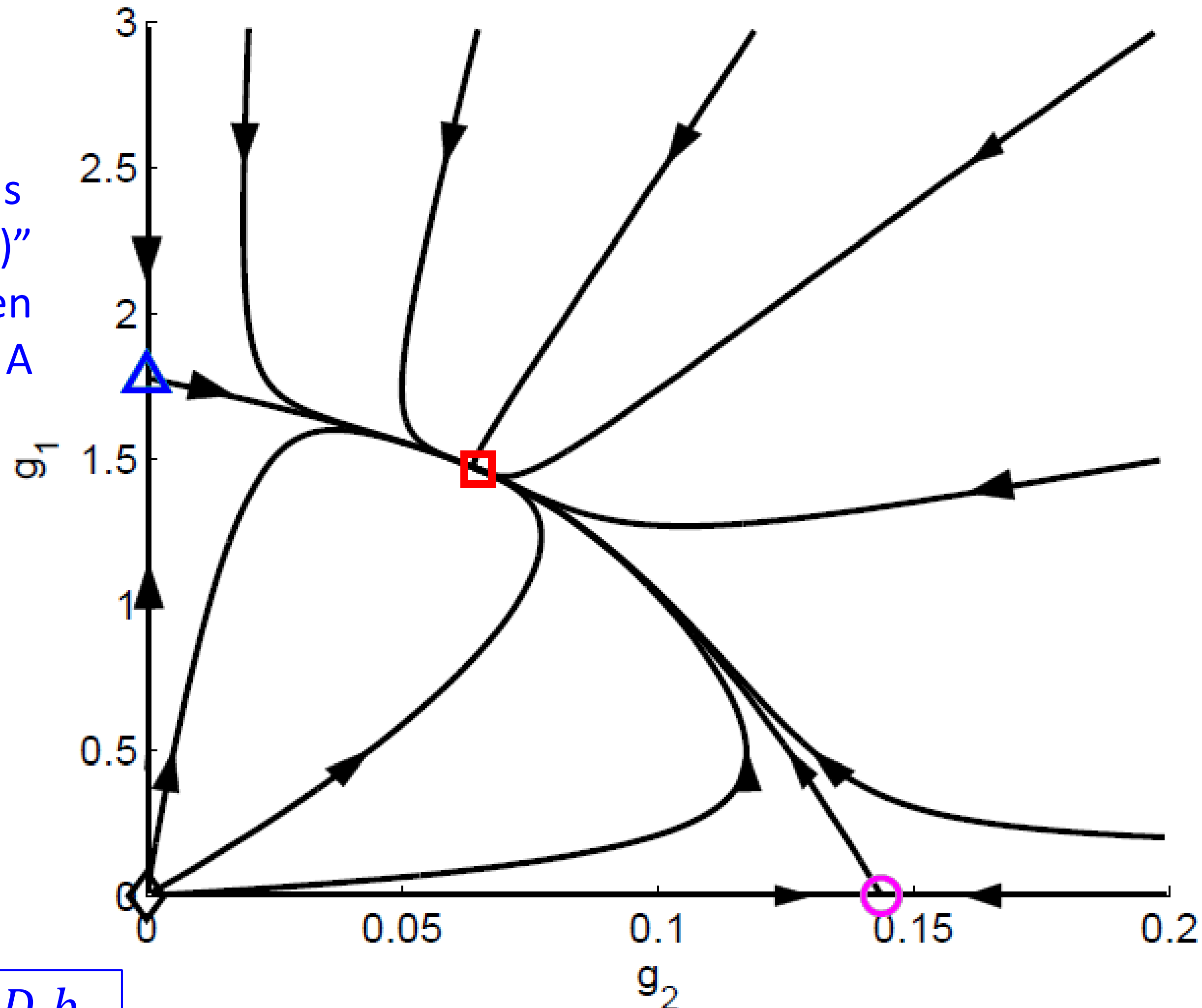

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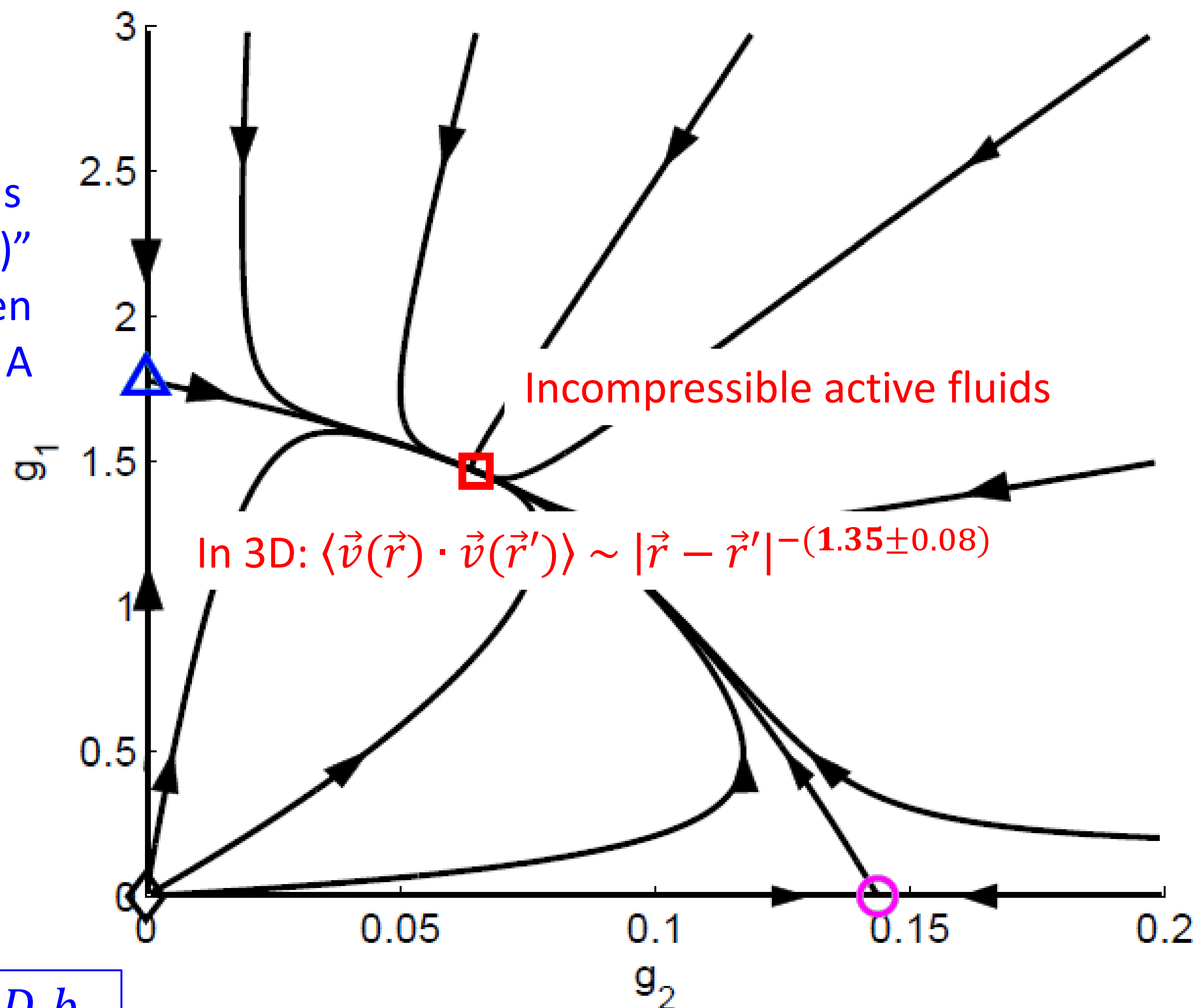

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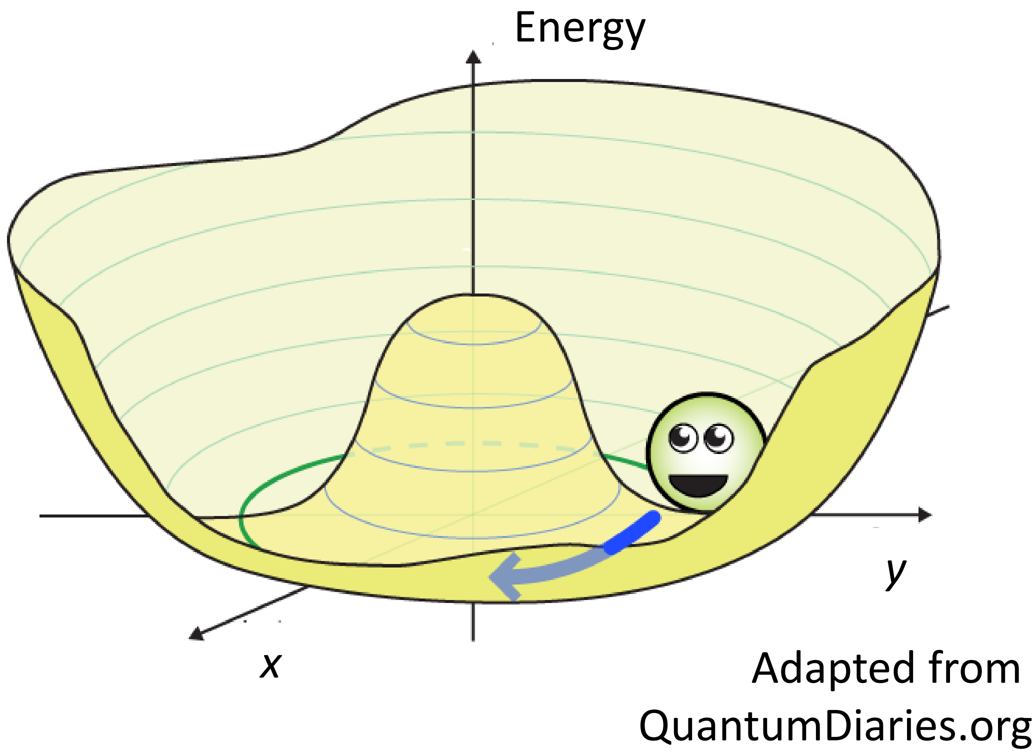
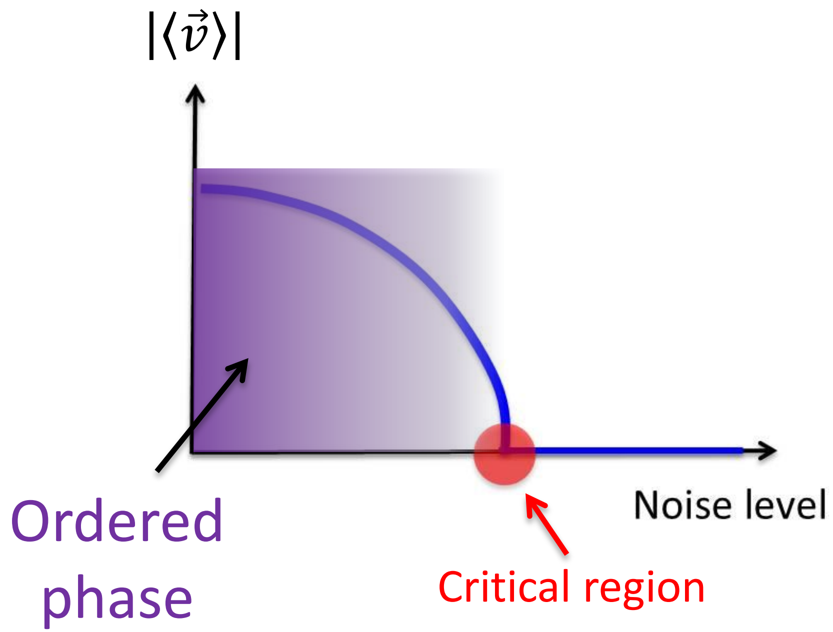


Ordered phase (2D)

Ordered incompressible active fluids



- **Universality is more than criticality!**
- Universal behaviour expected in the symmetry-broken phase of a continuous symmetry

Belitz, Kirkpatrick, Vojta (2005) Rev. Mod. Phys.

Ordered phase of 2D incompressible active fluids

EOM: $\partial_t \vec{v} + \vec{\nabla} P - \vec{f} = -\lambda(\vec{v}\cdot\vec{\nabla})\vec{v} - (a + bv^2)\vec{v} - \mu\nabla^2\vec{v} + cv^4\vec{v} + \xi(\nabla^2)^2\vec{v} + \cdots$

Ordered phase: $\mathbf{v} = (v_0 + u_x(\mathbf{r},t))\hat{x} + u_y(\mathbf{r},t)\hat{y}$

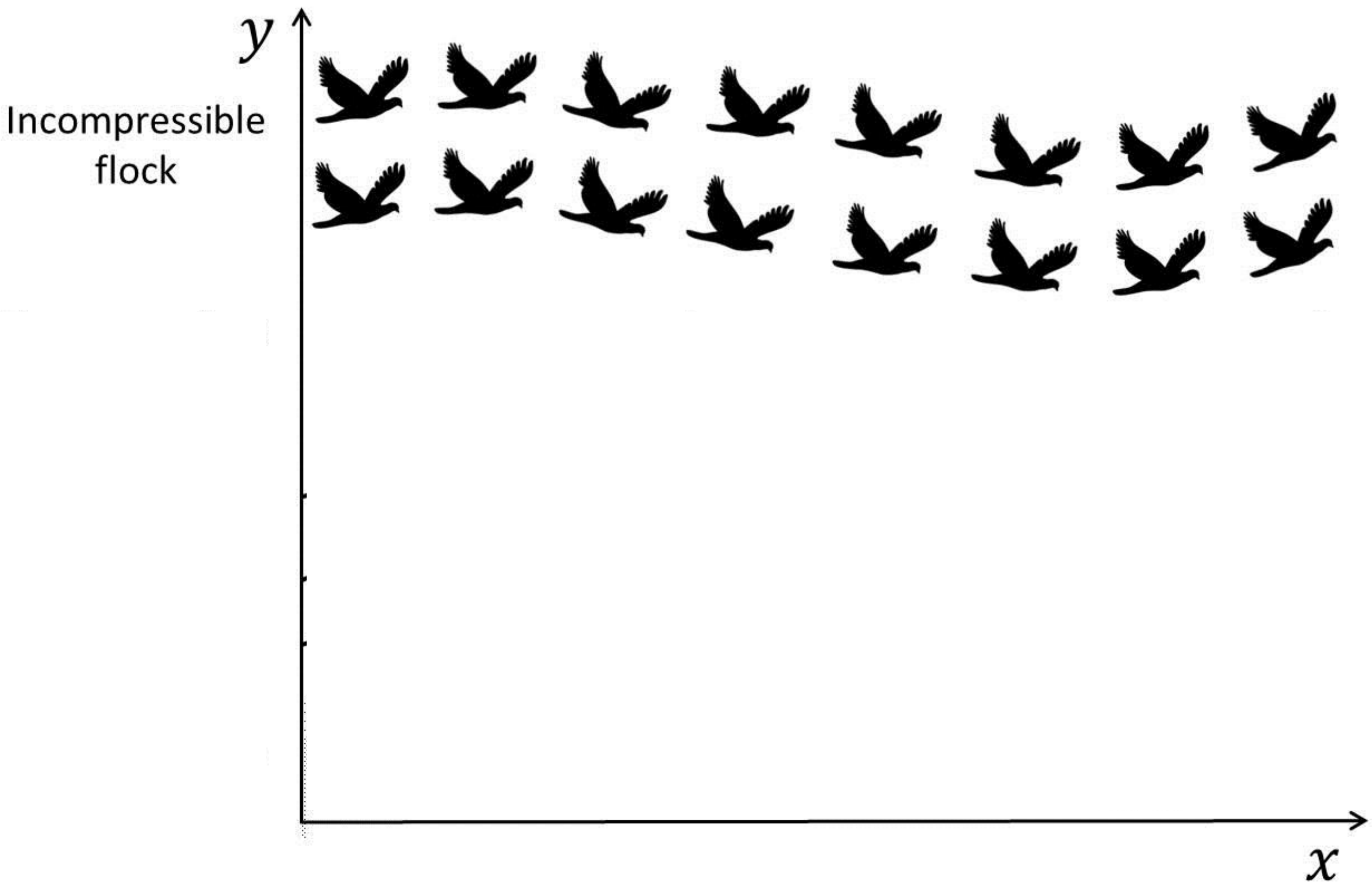
RG transformation

$$\partial_t u_x = -\partial_x P - 2b\left(u_x + \frac{u_y^2}{2v_0}\right) + \mu\nabla^2 u_x + f_x$$

$$\partial_t u_y = -\partial_y P - \frac{2b}{v_0}\left(u_x + \frac{u_y^2}{2v_0}\right) u_y + \mu\nabla^2 u_y + f_y$$

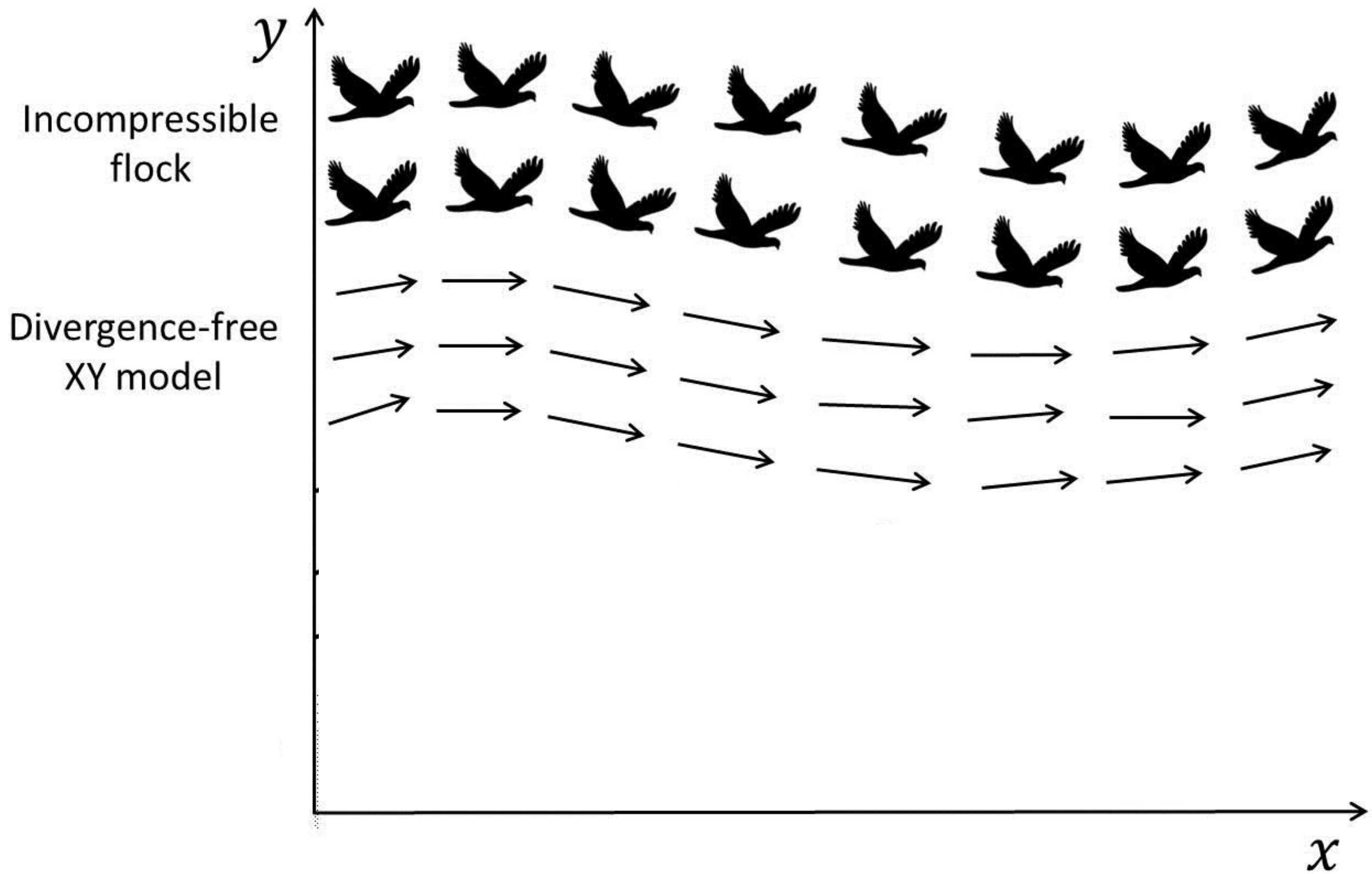
Level of coarse-graining

Universality of Kardar-Parisi-Zhang (KPZ)



Chen, Lee, Toner (2016)
Nature Comm. 7, 12215

Universality of Kardar-Parisi-Zhang (KPZ)



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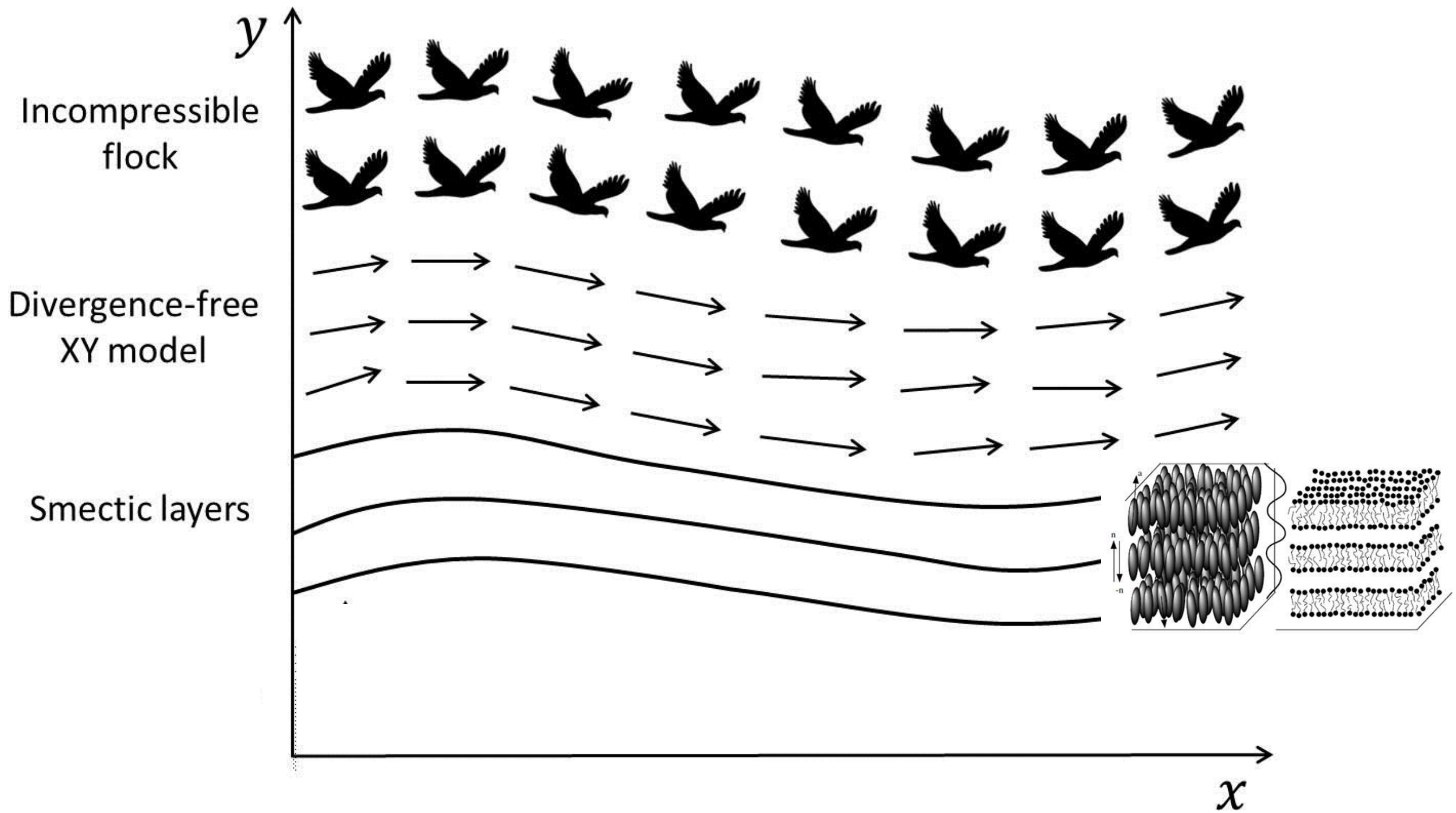

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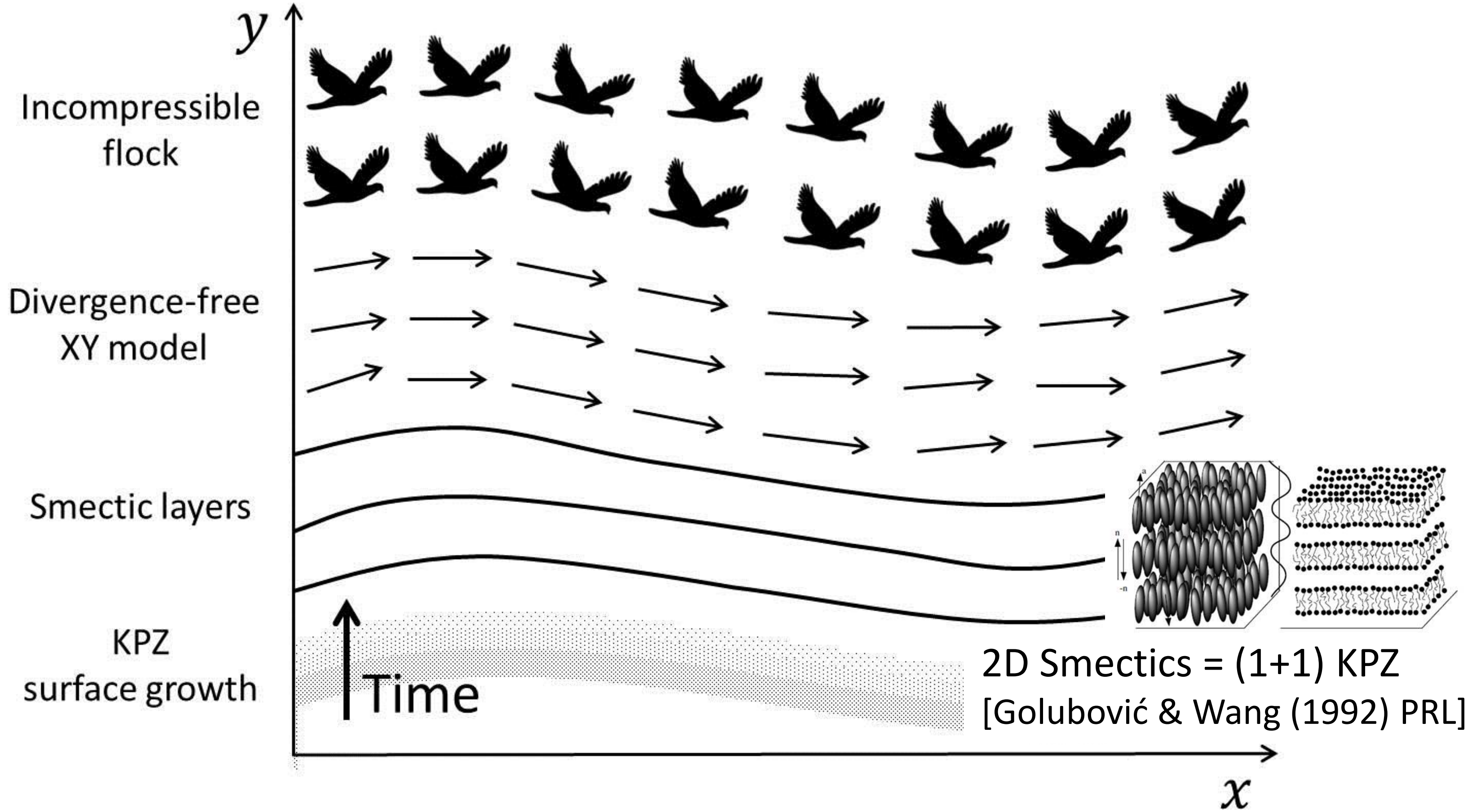

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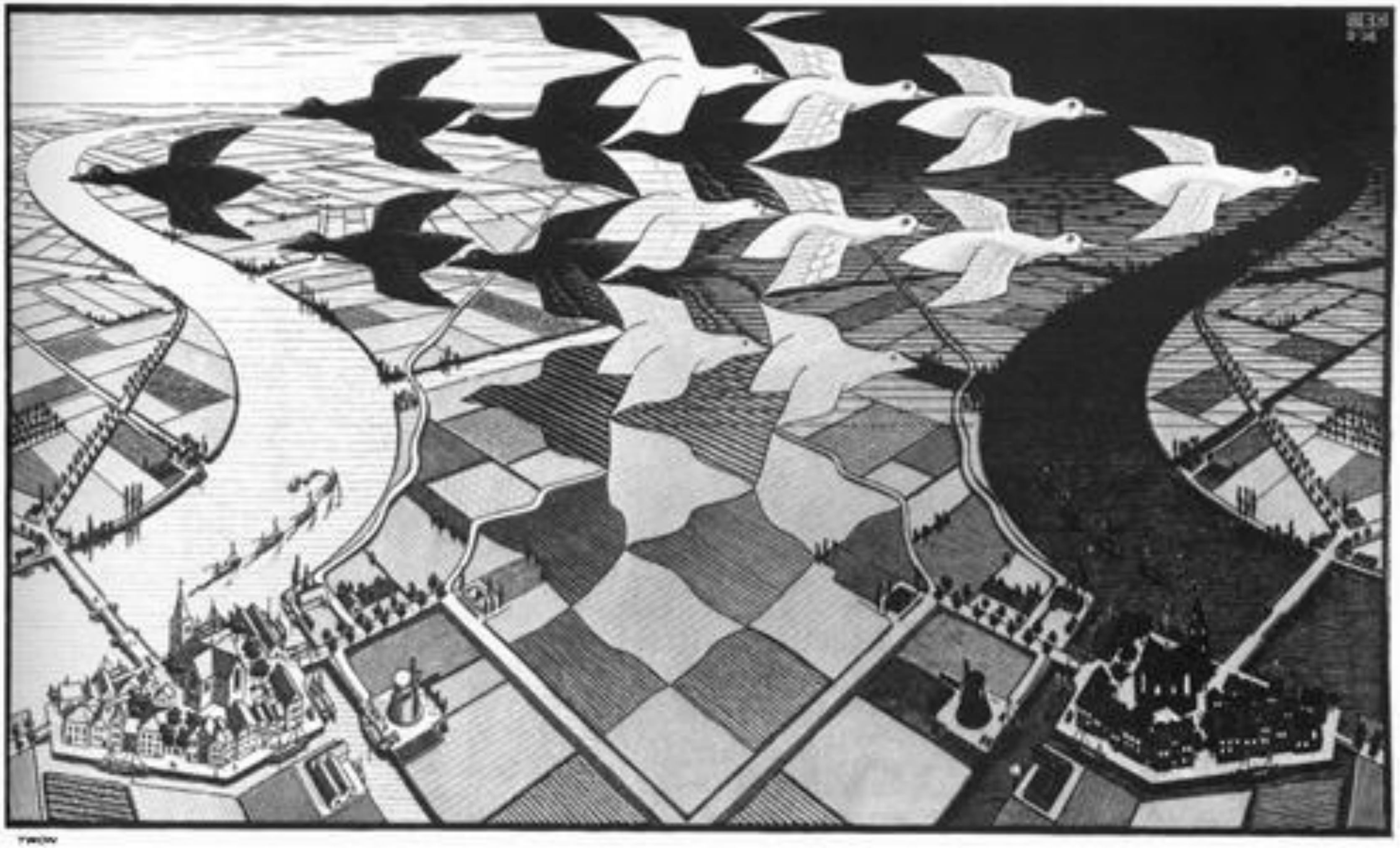
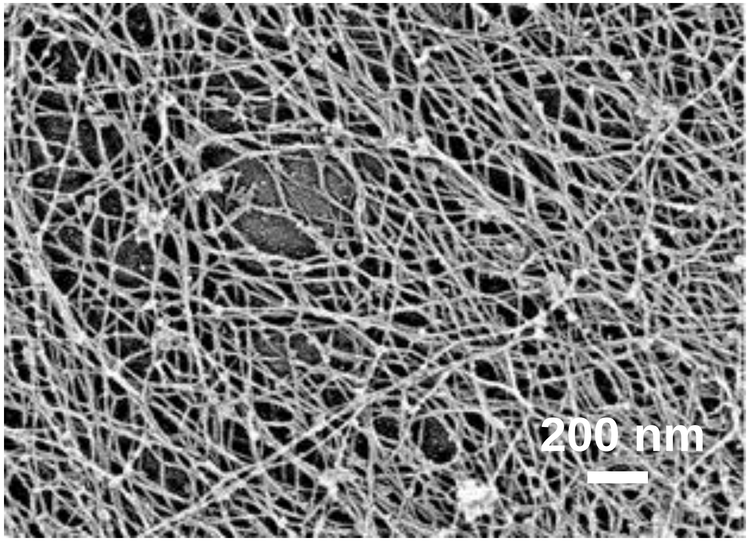
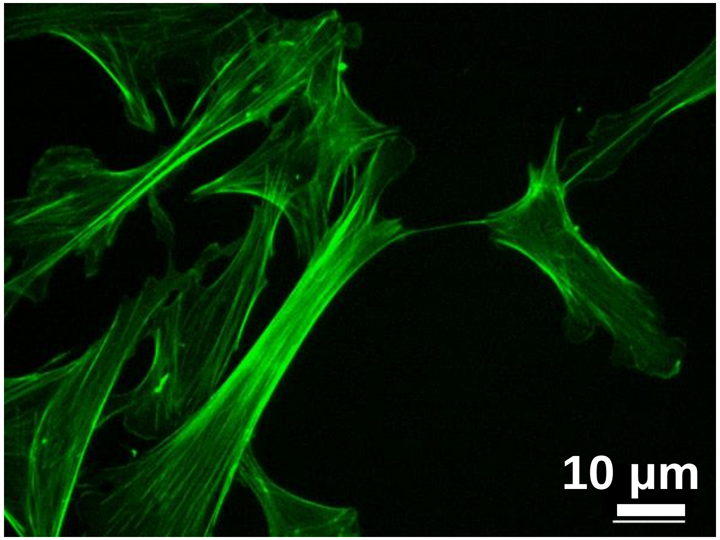
Nature Comm. 7, 12215



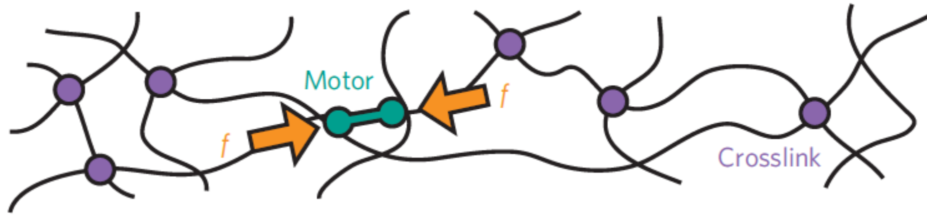
M.C. Escher (1938) *Day and Night*

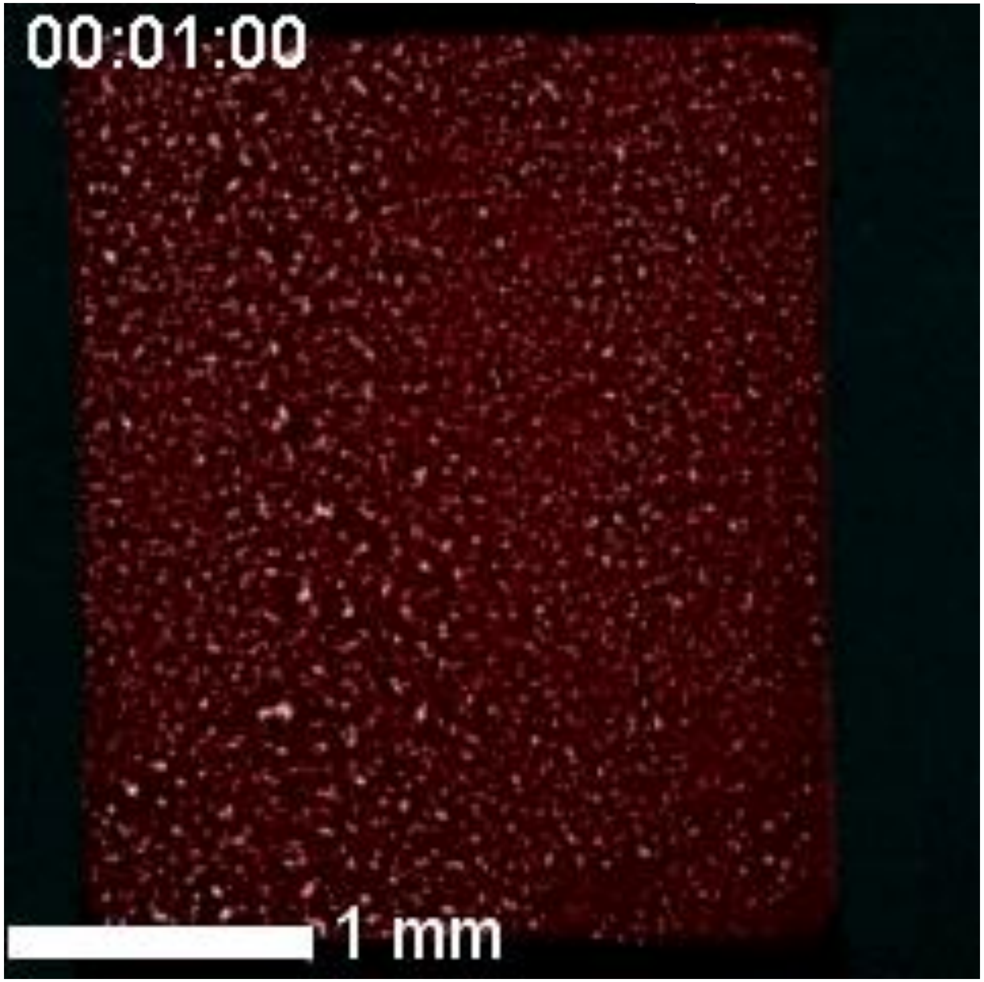


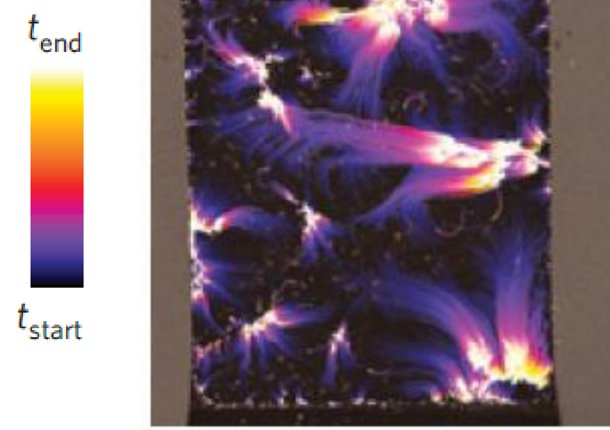


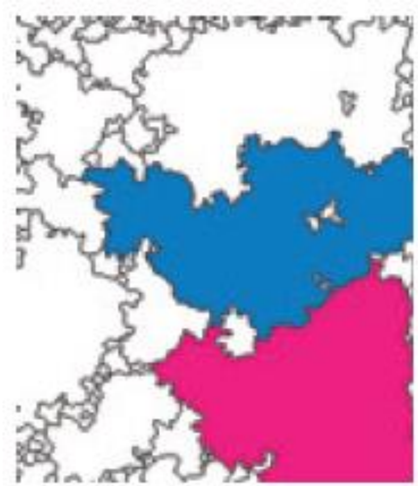
Active polymer networks

Active networks



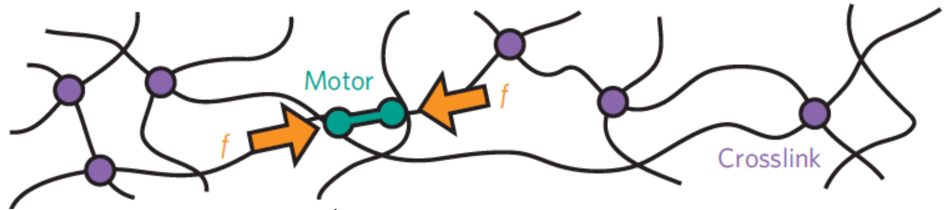


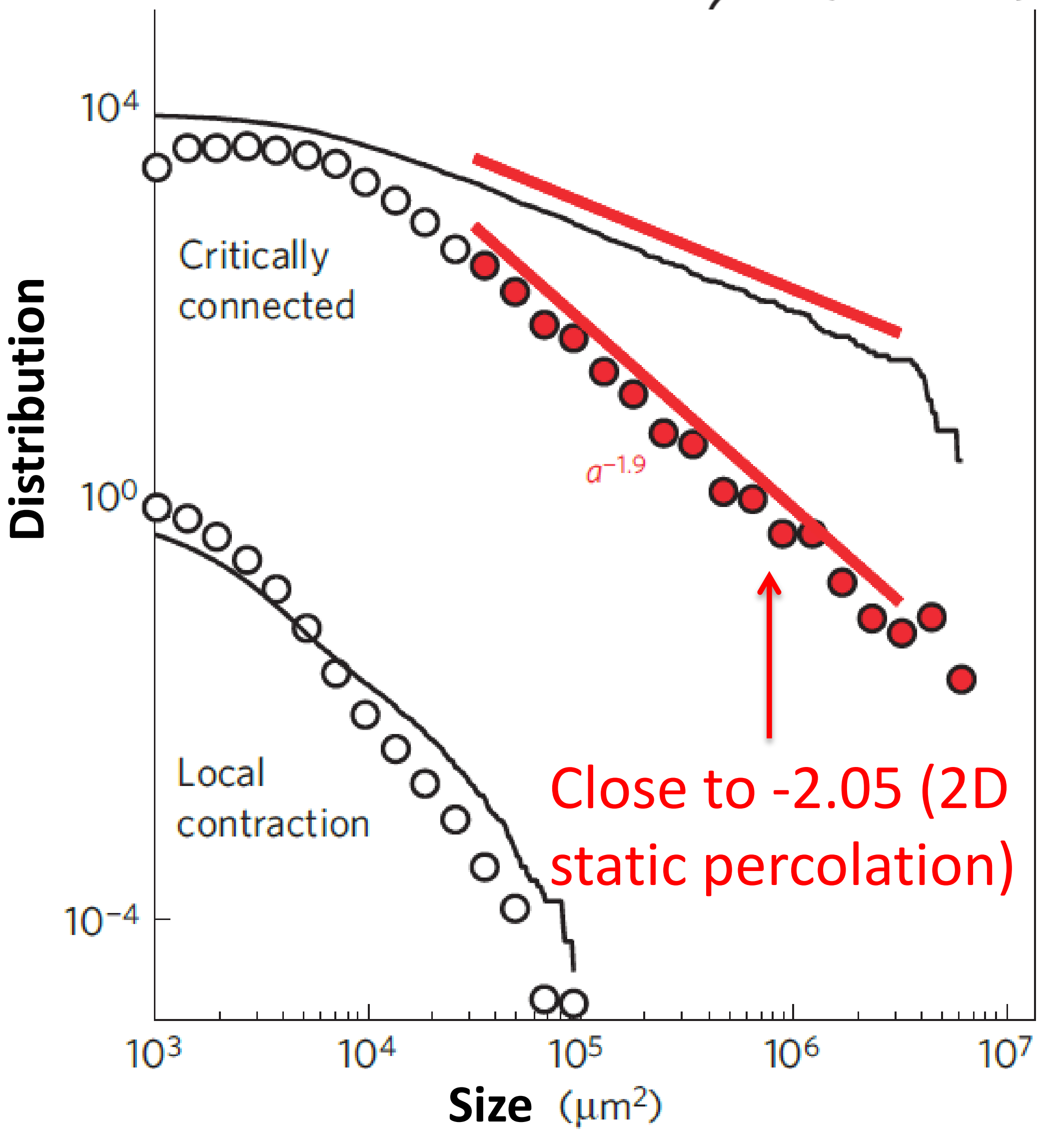


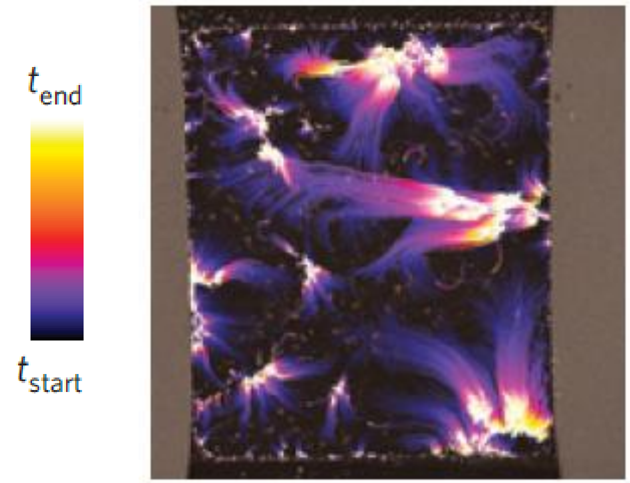


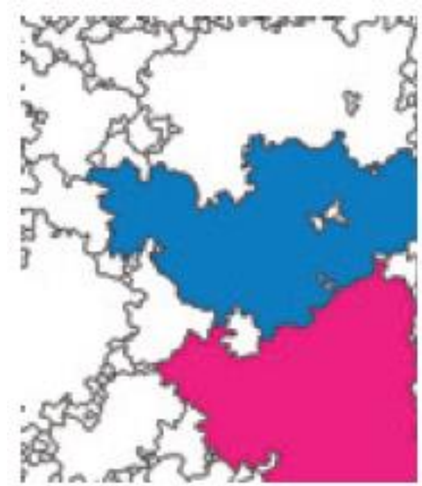
Alvarado et al. (2013)
Nature Physics

Active networks

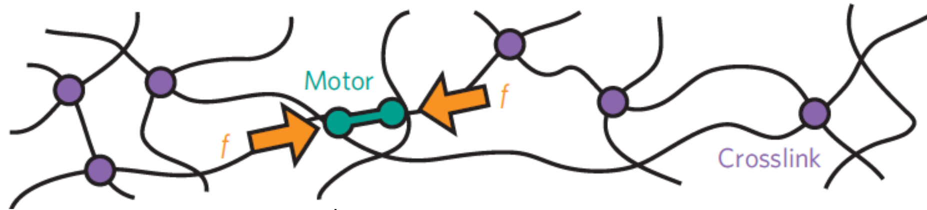


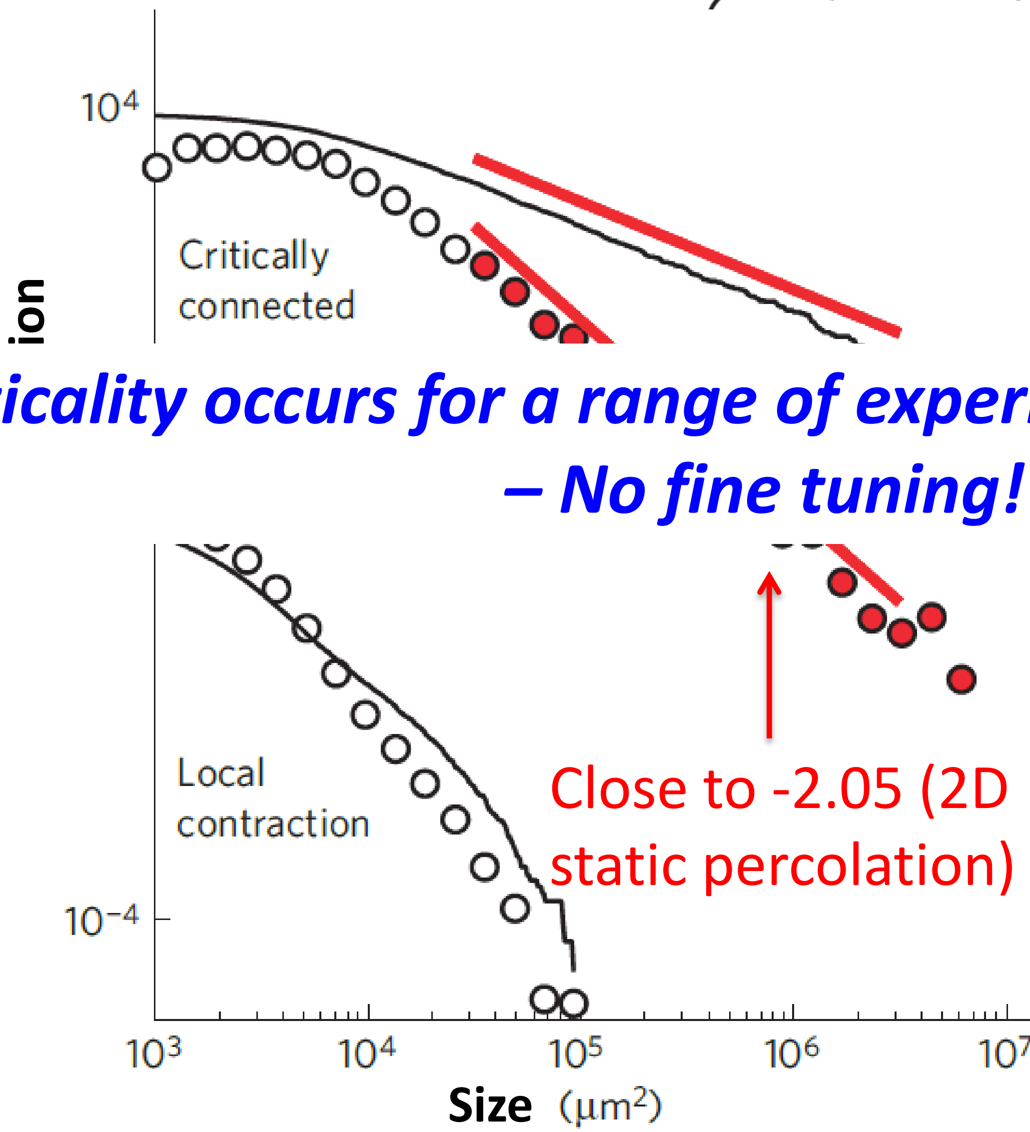




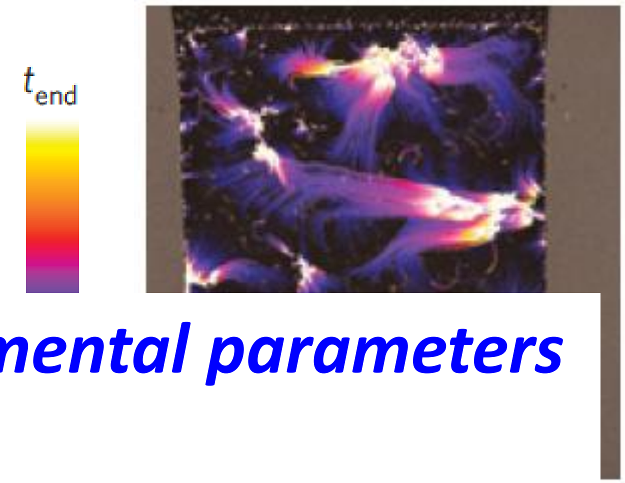


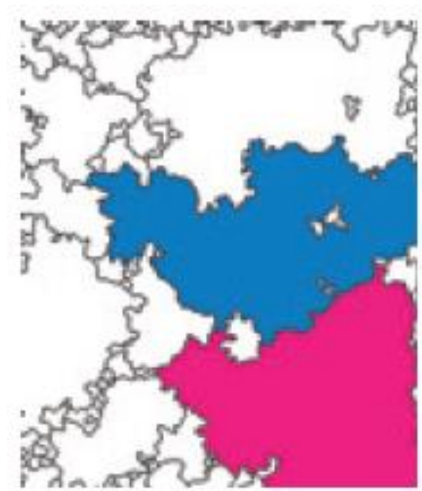
Active networks



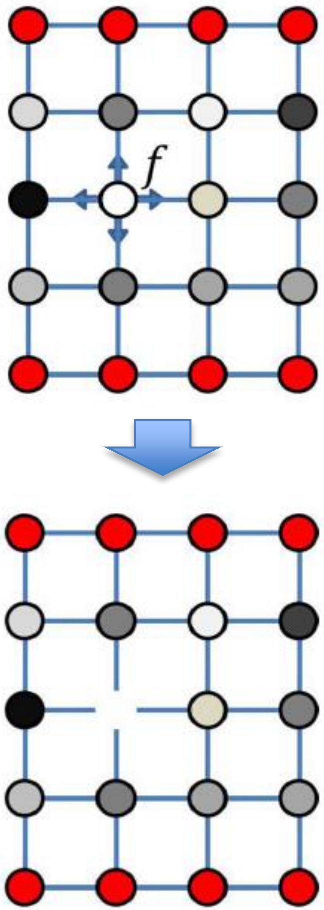


Criticality occurs for a range of experimental parameters – No fine tuning!





Minimal model: Random rupturing



Start with a fully connected squared lattice and delete interior nodes at random. If a connected cluster detaches from either boundary, then it is taken out of the system and its size recorded

Lee & Pruessner (2016) Phys. Rev. E

See also Sheinman, Sharma, Alvarado, Koenderink, and MacKintosh (2015) Phys. Rev. E; Sheinman, Sharma, Alvarado, Koenderink and MacKintosh (2015) Phys. Rev. Lett.; Pruessner and Lee (2016) Phys. Rev. Lett.

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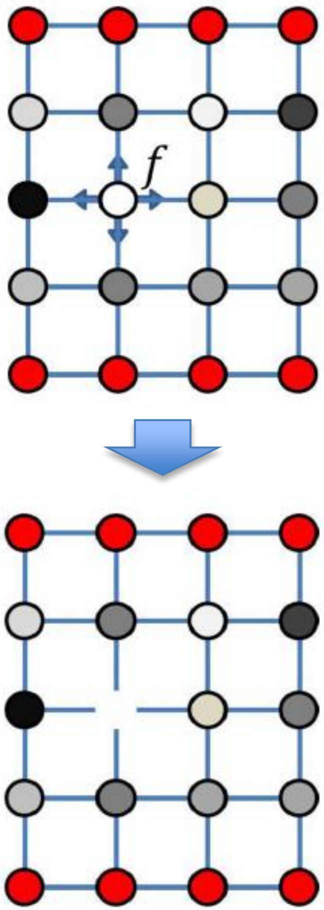
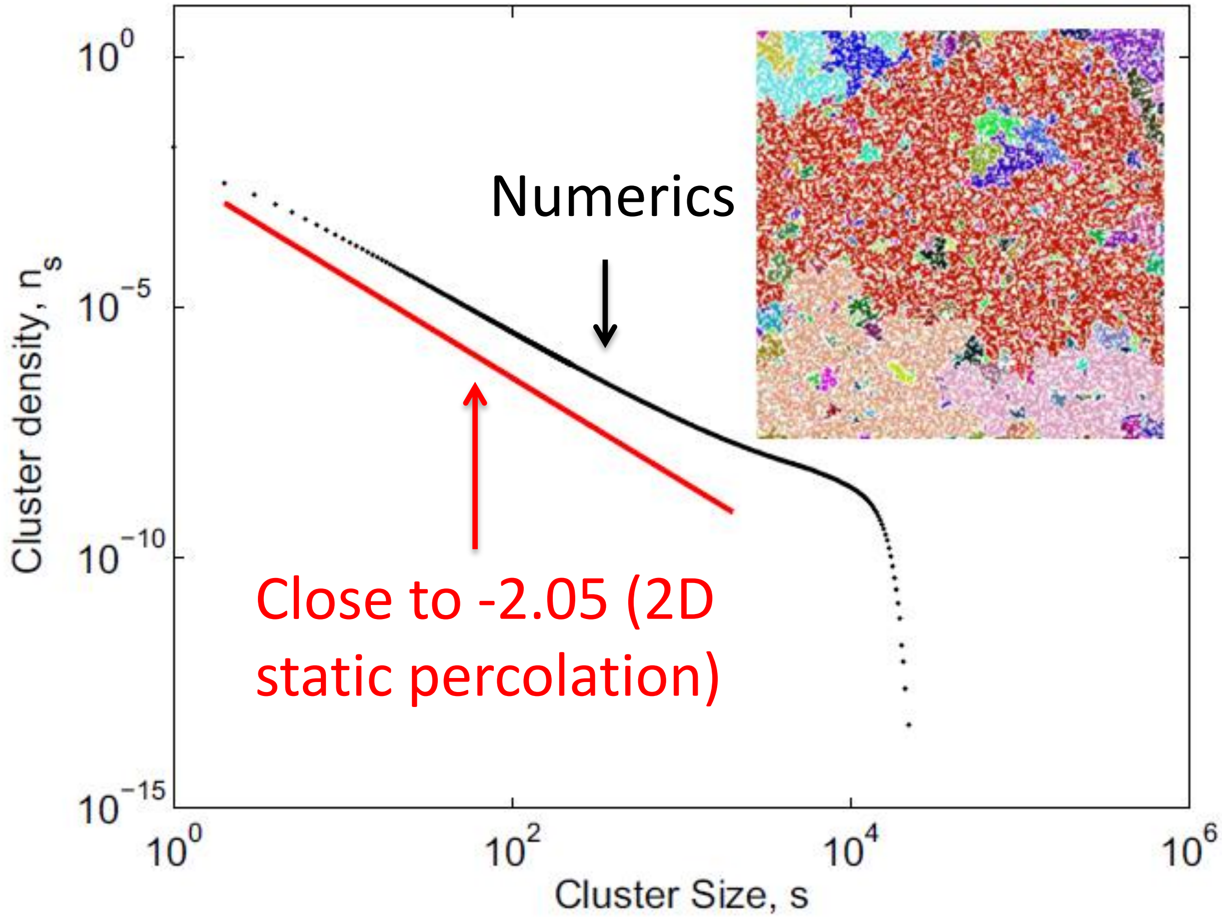


Lee & Pruessner (2016) Phys. Rev. E

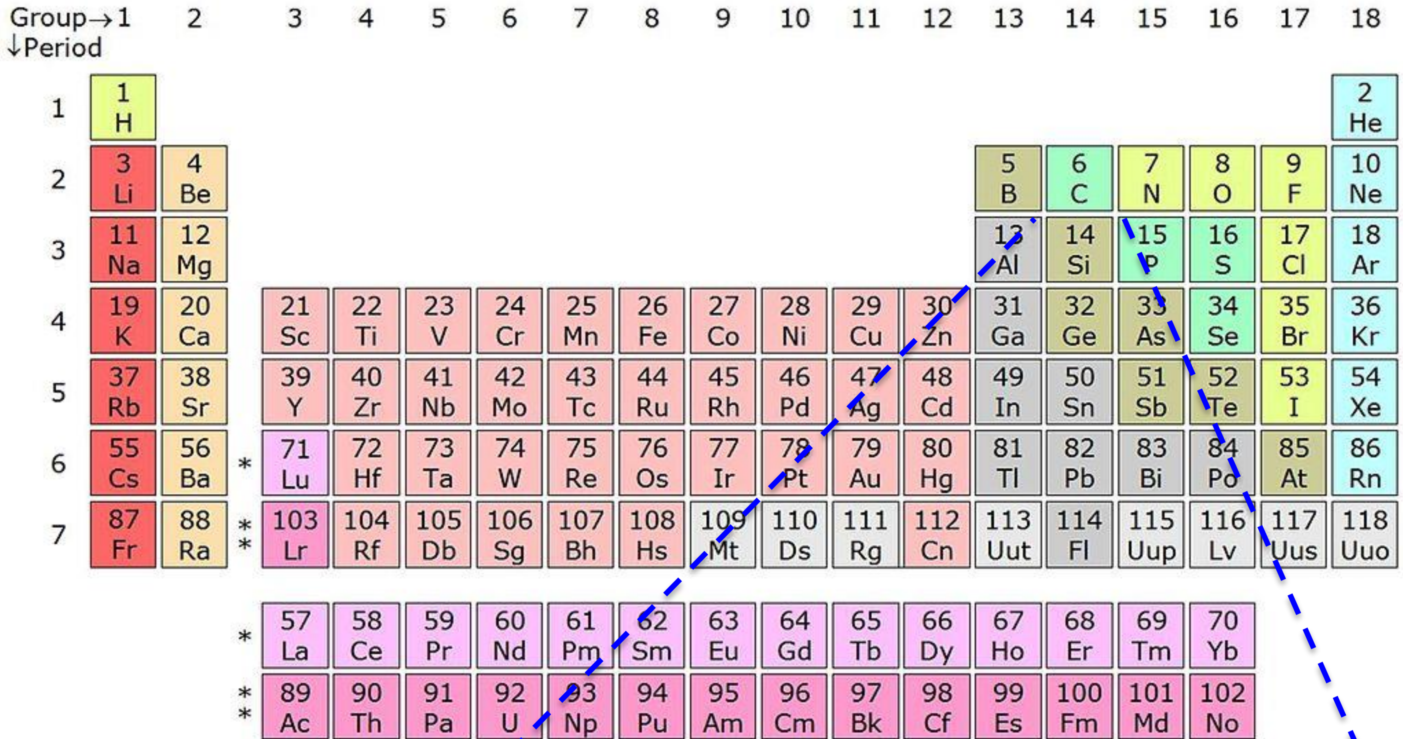


See also Sheinman, Sharma, Alvarado, Koenderink, and MacKintosh (2015) Phys. Rev. E; Sheinman, Sharma, Alvarado, Koenderink and MacKintosh (2015) Phys. Rev. Lett.; Pruessner and Lee (2016) Phys. Rev. Lett.

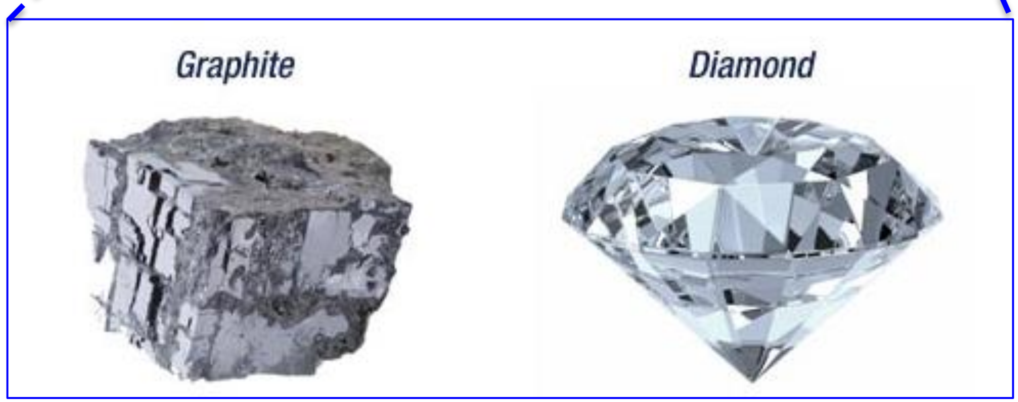
Active polymer networks

We do not have a dynamical theory of active polymer networks!

Outlook



Group→1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
↓Period
1 1 H 2 He
2 3 Li 4 Be 5 B 6 C 7 N 8 O 9 F 10 Ne
3 11 Na 12 Mg 13 Al 14 Si 15 P 16 S 17 Cl 18 Ar
4 19 K 20 Ca 21 Sc 22 Ti 23 V 24 Cr 25 Mn 26 Fe 27 Co 28 Ni 29 Cu 30 Zn 31 Ga 32 Ge 33 As 34 Se 35 Br 36 Kr
5 37 Rb 38 Sr 39 Y 40 Zr 41 Nb 42 Mo 43 Tc 44 Ru 45 Rh 46 Pd 47 Ag 48 Cd 49 In 50 Sn 51 Sb 52 Te 53 I 54 Xe
6 55 Cs 56 Ba * 71 Lu 72 Hf 73 Ta 74 W 75 Re 76 Os 77 Ir 78 Pt 79 Au 80 Hg 81 Tl 82 Pb 83 Bi 84 Po 85 At 86 Rn
7 87 Fr 88 Ra ** 103 Lr 104 Rf 105 Db 106 Sg 107 Bh 108 Hs 109 Mt 110 Ds 111 Rg 112 Cn 113 Uut 114 Fl 115 Uup 116 Lv 117 Uus 118 Uuo
* 57 La 58 Ce 59 Pr 60 Nd 61 Pm 62 Sm 63 Eu 64 Gd 65 Tb 66 Dy 67 Ho 68 Er 69 Tm 70 Yb
** 89 Ac 90 Th 91 Pa 92 U 93 Np 94 Pu 95 Am 96 Cm 97 Bk 98 Cf 99 Es 100 Fm 101 Md 102 No



Graphite
Diamond

‘Periodic table’ of universality classes

Equilibrium

Designation	System
A	Kinetic Ising anisotropic magnets
B	Kinetic Ising uniaxial ferromagnet
C	Anisotropic magnets structural transition
H	Gas–liquid binary fluid
E	Easy-plane magnet, $h_z = 0$
F	Easy-plane magnet, $h_z \neq 0$ superfluid helium
G	Heisenberg antiferromagnet
J	Heisenberg ferromagnet

Hohenberg and Halperin (1977) “Theory of dynamic critical phenomena” Rev. Mod. Phys.

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Non-equilibrium

Randomly stirred fluids
[Forster, Nelson & Stephen (1977) Phys. Rev. A]

Directed percolation
[Kinzel (1983)]

Kardar-Parisi-Zhang
[KPZ (1983) Phys. Rev. Lett.]

Living matter

Incompressible active polar fluids at criticality
[Chen, Toner, Lee (2015) New J. Phys.]

Universalities in living matter

Designation	System	Biological systems	Transition	Ordered phase
A	Kinetic Ising anisotropic magnets	?	?	?
B	Kinetic Ising uniaxial ferromagnet	?	?	?
C	Anisotropic magnets structural transition	?	?	?
H	Gas–liquid binary fluid	Motile bacteria	?	Lifshitz-Slyozov
E	Easy-plane magnet, $h_z = 0$	Epithelial tissue	IAF	KPZ
F	Easy-plane magnet, $h_z \neq 0$ superfluid helium	?	?	?
G	Heisenberg antiferromagnet	?	?	?
J	Heisenberg ferromagnet	Birds	IAF	?

Summary

- Universality is everywhere in biology
- Critical order-disorder transition
  - A novel universality class
- Ordered phase (2D)
  - Kardar-Parisi-Zhang universality class
- Active polymer networks (2D)
  - Static percolation universality class
- Outlook: Characterising of living matter

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Thank you for your Attention!